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Simple Qualitative Assessment of Investment Recommendations

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ABSTRACT: This paper outlines a simple methodology for assessing "invest/don't invest" recommendations on portfoliostrategies, made by professionals offering 'expert' investment consulting servicesto large institutional investors.Recommendations are compared to performance of portfolios of fixed income, in reference to proxies ofbenchmarks. Estimation techniques for such task abound across regulatory and risk-management applications in the finance industry. The same techniques may serve as blueprint toward the development of applied practices, through which indications about performance forthe 'expert' consultant services themselves, could be obtained.The use of such practices in the investment industry, could soon be required bythe U.S. financial regulators.

KEYWORDS: *Principal Components, Discriminant Analysis, Information Ratio, Pay-to-Play*

I.INTRODUCTION

The term "pay-to-play" has emerged as a concern of U.S. regulatory authorities regarding investment advising, as it relates torecommending portfolios to institutionalclients. It refers to advisers' encouraging or receiving monetary benefit from a portfolio manager, in exchange for access tothe advisers' client-investors. The advisers' investment recommendations increase flow of investor assets into portfolios managed by engaging investment firms.The U.S regulators investigate payments by various methods imposed on money management firms, in exchange for access to advisors' clients [1].A better method for reinforcing the fiduciary responsibility of investment advisers comes from the newly reincarnated 'Fiduciary Rule' proposed by the U.S. Department of Labor (as of the writing of this paper). A full account of the history of this rule is beyond scope. It can be argued, that the original Dodd-Frank (2010) regulatory framework did not take into consideration the systemic exposure created by these retirement service practices. The Federal Reserve's Comprehensive Capital Analysis and Review (CCAR), for example, looks at capital levels of the nation's largest banks. Dodd-Frank Act Stress Tests (DFAST), on the other hand, impose a "forward-looking, quantitative evaluation of bank capital that demonstrates how a hypothetical set of stressful economic conditions […] affect the capital ratios of large firms" [2]. Thus, regulatory authorities have historically focused on the financial institutions. Another intermediary industry is becoming important, as economic agents that lived through the 2008 real estate related crisis are reaching retirement age.

Recommendations split portfolios into categories of (i) invest and (ii) do not invest. Consultants have historically resisted simple quality-control on their 'expert' advise.What appears as challenging, is that categories (i) and (ii) are 'qualitative' whereas performance of portfolios is measured quantitatively. Discriminant analysis is one of the simplest methods that can be used, to assess the short-term impact of investment recommendations on the welfare of institutional investors. Such impact comes as contemporaneous improvement or loss in information ratio, as a result of following investor recommendations. The actual efficacy of these recommendations, the motives of plan sponsors in following them, and the impact on the institutional investor lie outside the scope of this analysis. The analysis only examines the correspondence of recommendations to performance of strategies recommended at the time the recommendations are observed. These recommendations are'forward-looking' attempts to capture the expected probability of outperformance of a strategy. Consultants form and publish non-quantified opinions about this probabilityafter meetings and communications with the rated investment managers. Internal processes result in such ratings, implying the binary recommendations. Ratings are limited in scope to an assessment of the overall investment decision-making process of portfolio managers. Still, institutional investors may be interested to know the correspondence of contemporaneous performance of a strategy to the recommendations issued through rating. This proposed methodology quantifiesprobability of outperformance and relates it to a hyperplane-separation of strategies by rating, all based on the sensitivity of portfolios to a market force that is relevant to fixed income, such as swap rates. The procedure suggesta standard against which consultant ‘expertise’ could be assessed. It offers hints fora direction that research efforts could take, in attempts to suggest areas of concern to regulators.

II.GENERAL PROCEDURE

The comparison of strategy ratings against portfolio performance takes place in two stages, the results of which are compared. Ahead of the comparison, the sensitivity of portfolios to principal components of the swap curve is estimated and resulting coefficients are used as independent variables [3]. The first stage shows probability of out-performance as a function of sensitivity of portfolios to market variable components. The second stage separates recommended from non-recommended strategies on a *Z*-score that is a function of these same coefficients. Since probability estimated in the first stage is representative of expected outperformance, and the discriminant score in stage two separates strategies adequately, one would like to see a positive relation between thesetwo. At the very least that relation should not be strictly negative. The investor should not suffer an immediate loss in performance in the short-run, as a result of adhering to ‘forward-looking’ consultant recommendations. The underlying market variables, which probability of outperformance and ratings-based discriminant *Z*-scores are based on, are LIBOR swap rates as explained below. Deviations between outperformance and recommendationareexplained in terms of swap rates. This process helps uncover elements in consulting, where direct attention to concrete research views on a whole fixed income investment universe is warranted. The comparison between outperformance probability and rating *Z*-scores can also be extrapolated to portfolios un-examined by consultants, to gauge *a-priori* the kind of recommendation that unrated strategies may fall into. Probabilities for outperformance and discriminant *Z*-scores for equities, international, or emerging markets, can be estimated by changing underlying market variables.

III.RECOMMENDATIONS AND OUTPERFORMANCE

The method employs the techniques of: (i) Simple Linear Regression, (ii) Logistic Regression, (iii) Discriminant Analysis, and (iv) Principal Component Analysis. For example, portfolio outperformance is gauged by principal components of most common benchmarks in a universe, instead of ones stated by each strategist. Since benchmark components account for the largest portion of all index variability, this method of measurement minimizes ‘alpha’ to a level that is not explained by the market. It is thus, mostly ‘skill’. The returns of each portfolio are regressed against the four largest components to find such benchmark proxies. Portfolio returns in excess of the proxies are assumed to be generated through the active management pursuit on total performance,associated withprincipal components of current and lagged swap rates. The resulting excess returns determine the information ratio (IR) for portfolios. Information ratios are logit-regressed against beta-coefficients of respective portfolios toprincipal components of the swap curve, to estimate the probability of outperformance. A discriminant function, also based on portfolio beta-coefficients to the swap rate components,determines separation between recommended and non-recommended strategies. To the degree that the classificationis robust, in reference to sensitivities of respective portfolios to components of swap rates, the resulting discriminant functions adequately separate strategies into ones with positive, and with negative *Z*-scores. The estimated probability through logistic regression is simply compared to the discriminant score based on rating.Table 1 shows condensed data for tenors of the 36 variables.

Table 1: Current and Lagged Values of U.S. LIBOR Swap Rates

				Current Swap Rates					
Date	**1M**	**3M**	**6M**	**1Y**	**5Y**	**10Y**	**15Y**	**…**	**50Y**
10/1/2013	0.1740	0.2431	0.3662	0.6271	1.6040	2.8440	3.3540		3.7175
9/1/2013	0.1788	0.2489	0.3685	0.6294	1.5400	2.7660	3.2870		3.6660
8/1/2013	0.1821	0.2595	0.3930	0.6671	1.8100	2.9860	3.4440		3.7250
…	…	…	…	…	…	…	…		…
9/1/2005	3.8637	4.0650	4.2306	4.4400	4.6550	4.7900	4.9180	…	5.0440
				Swap Rates Lagged (L) One Month					
Date	**L 1M**	**L 3M**	**L 6M**	**L 1Y**	**L 5Y**	**L 10Y**	**L 15Y**	**…**	**L 50Y**
10/1/2013	0.1788	0.2489	0.3685	0.6294	1.5400	2.7660	3.2870		3.6660
9/1/2013	0.1821	0.2595	0.3930	0.6671	1.8100	2.9860	3.4440		3.7250
8/1/2013	0.1867	0.2656	0.3965	0.6732	1.5560	2.7720	3.2840		3.6200
…	…	…	…	…	…	…	…		…
10/1/2005	3.8637	4.0650	4.2306	4.4400	4.6550	4.7900	4.9180	…	5.0440
				Swap Rates Lagged (L L) Two Months					
Date	**L L 1M**	**L L 3M**	**L L 6M**	**L L 1Y**	**L L 5Y**	**L L 10Y**	**L L 15Y**	**…**	**L L 50Y**
10/1/2013	0.1821	0.2595	0.3930	0.6671	1.8100	2.9860	3.4440		3.7250
9/1/2013	0.1867	0.2656	0.3965	0.6732	1.5560	2.7720	3.2840		3.6200
8/1/2013	0.1946	0.2731	0.4134	0.6856	1.5680	2.7020	3.1722		3.4540
…	…	…	…	…	…	…	…		…
11/1/2005	3.8637	4.0650	4.2306	4.4400	4.6550	4.7900	4.9180	…	5.0440

Analyses of investment management performance rely on excess (active) returns of a strategy over an index that is the benchmark. Both a benchmark and the strategy are affected by underlying market forces, comprising current and lagged monthly rates of twelve points on the U.S. LIBOR swap rate curve. Current, as well as lagged values of the swap curve have an effect on the performance of fixed-income portfolios. Thus, the same twelve points on the curve, lagged one month, and one month more, are used. In total, 36 variables describe the effects of underlying market forces on active portfolio returns. The U.S. LIBOR ratesthat adequately describe relevant phenomena are for one, three and six-month, as well as the one, five, 10, 15, 20, 25, 30, 40 and 50-year swap rates;and the same tenors lagged once; and once again. Variable-reduction through principal components assists in distilling just four effects that capture more than 99% of the variability in all the 36 variables discussed (see Table 2).Decomposition of the covariance matrix between thirty-six variables, over monthly returns between 11/2005 and 10/2013, allows for selection of the linear combinations of variables that account for the largest part of overall rate variability.

Table 2: Eigenvectors of Covariance Matrix of Current and (L)Lagged Swap Rates

Tenor	Swap Rates Up for Quarter	Swap Rates Flat for Quarter	Current Down LLagged Up	Persistent Quant. Easing	Current Flat LLagged Steep	Long End Down, Up, Down	Short End Down, Up, Down
1M	24%	15%	-15%	31%	37%	-5%	-29%
3M	24%	17%	-12%	9%	32%	-8%	-19%
6M	23%	17%	-14%	-11%	28%	-4%	-18%
1Y	21%	14%	-17%	-24%	30%	-3%	-16%
5Y	18%	-11%	-21%	-31%	-3%	-11%	0%
10Y	14%	-17%	-21%	-13%	-11%	-11%	5%
15Y	12%	-18%	-22%	-2%	-13%	-14%	4%
20Y	11%	-18%	-22%	5%	-13%	-15%	2%
25Y	11%	-18%	-23%	8%	-12%	-15%	1%
30Y	11%	-18%	-23%	10%	-12%	-15%	1%
40Y	10%	-18%	-23%	10%	-12%	-16%	3%
50Y	11%	-18%	-23%	10%	-12%	-16%	2%
L 1M	24%	16%	1%	34%	4%	1%	50%
L 3M	24%	19%	1%	10%	-2%	5%	41%
L 6M	23%	18%	-1%	-9%	-2%	7%	36%
L 1Y	21%	15%	-5%	-23%	6%	9%	31%
L 5Y	18%	-11%	1%	-31%	1%	25%	4%
L 10Y	14%	-17%	4%	-12%	-1%	30%	-1%
L 15Y	12%	-18%	2%	-1%	0%	29%	-3%
L 20Y	11%	-18%	0%	6%	1%	28%	-4%
L 25Y	11%	-18%	0%	10%	3%	28%	-5%
L 30Y	11%	-18%	0%	11%	3%	28%	-5%
L 40Y	11%	-18%	0%	12%	3%	28%	-6%
L 50Y	11%	-18%	-1%	12%	3%	28%	-6%
LL 1M	24%	18%	16%	32%	-35%	-1%	-23%
LL 3M	24%	20%	14%	9%	-37%	1%	-21%
LL 6M	23%	19%	10%	-9%	-34%	1%	-18%
LL 1Y	21%	16%	7%	-21%	-20%	0%	-17%
LL 5Y	18%	-9%	22%	-32%	-2%	-14%	-3%
LL 10Y	14%	-15%	26%	-15%	3%	-18%	-1%
LL 15Y	12%	-16%	25%	-4%	7%	-16%	1%
LL 20Y	11%	-16%	24%	3%	9%	-15%	2%
LL 25Y	11%	-16%	24%	7%	11%	-15%	3%
LL 30Y	11%	-16%	23%	9%	11%	-15%	2%
LL 40Y	11%	-16%	23%	9%	12%	-15%	3%
LL 50Y	11%	-16%	23%	9%	12%	-14%	3%
Variance:	*92.43%*	*97.98%*	*98.87%*	*99.29%*			

Four out of possible 36 components account for 99.29% of the variability in swap rates over the selected period. Challenges in this reduction in the number of variables are: (i) discerning the kind of economic phenomena that linear combinations of original swap rates describe, and (ii) translating components into original variables. As an example, the values of the first principal component between 10/1/2013 and 11/1/2005 are calculated as follows:

$$(24\%)0.1740 + (24\%)0.2431 + (23\%)0.3662 + \cdots + (11\%)3.7250 = 10.2017$$
$$(24\%)0.1788 + (24\%)0.2489 + (23\%)0.3685 + \cdots + (11\%)3.6660 = 10.1498$$
$$(24\%)0.1821 + (24\%)0.2595 + (23\%)0.3930 + \cdots + (11\%)3.4540 = 10.0542$$
$$(\ldots)$$
$$(24\%)3.8637 + (24\%)4.0650 + (23\%)4.2306 + \cdots + (11\%)4.9180 = 26.8003$$

The weights in parentheses, from the first column in Table 2, are multiplied with swap rates in Table 1 and then summed. In order to derive values for the rest of the components, the swap rates in Table 1 are simply multiplied with weights in subsequent columns in Table 2, and construct a different linear combination of rates each time.

The process of constructing linear combinations of swap rates, referred to as 'principal components of rates', is shown in Column A, Level 2 of the schematic representation in Fig. 1 below. Briefly, along Column A of Fig. 1, the swap rates are turned into components and used as explanatory variables for Outperformance Probability. The latter is estimated independently, as described in Column C, where it is the benchmark returns that are turned into principal components, instead. After arriving at returns of the proxy benchmark (Column B, Level 3), the resulting information ratio is converted to probability (Column A, Level 4). That probability is re-estimated with portfolio coefficients to swap rate components and is compared to the discriminant *Z*-score.Returning to challenges (i) and (ii) above, describing the rate components is somewhat arbitrary. For example, the weights in the fourth column in Table 2 correspond to a phenomenon in which 1-year through 15-year swap rates go down (they are multiplied by a negative weight) in current and lagged. One could associate this phenomenon with the post-crisis experience in U.S. rates, in which quantitative easing (QE) has supported the prices of financial instrument in the middle of the swap curve, keeping yields in that range relatively low.Consultants discuss QE, but rarely if ever, quantify it. For the translation of the components back into original rates, the value of a component that is not changed would have to be set at its current or long-run average while finding the curve effect of a component shock. For example, given that the standard deviation of the component '*Swap Rates Up for the Quarter*' is 8.74, a positive move at the 5% level of significance from current levels would amount to a value of 24.63. All other components would stay at their current (10/2013) values. Component values would be multiplying by the inverse of Table 2.

$$PC1_{10/2013} + 1.65\sigma_{PC1} = 10.2017 + 1.6500 \times 8.7432 = 24.6279$$

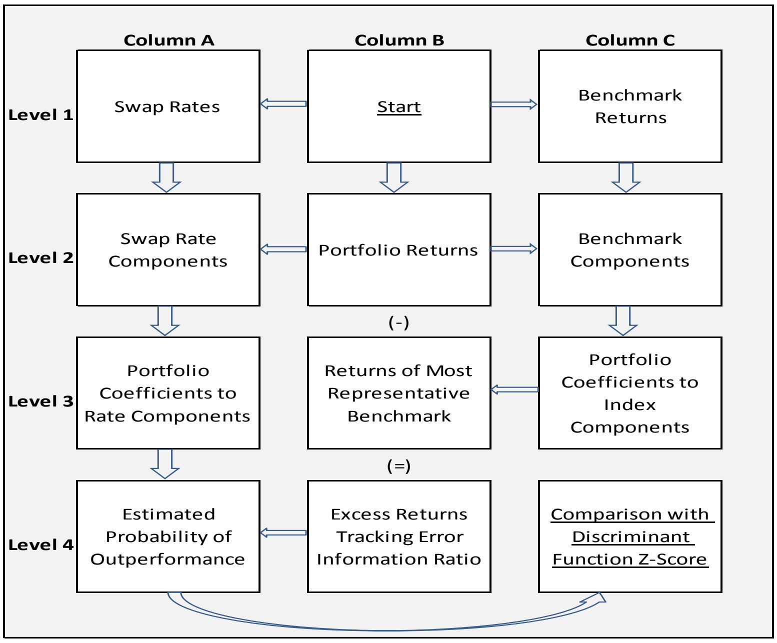

Figure 1: schematic representation of estimated outperformance probability

IV.MOST REPRESENTATIVE BENCHMARK: COMPONENTS OF INDICES

It is often the case that the stated benchmark for a strategy is not representative of the universe of portfolios. That fact allows a portion of beta to 'seep' into alpha. As discussed previously, the presented method alleviates some of this 'excess alpha' issue by finding a linear combination of benchmarks that captures most of the effects of the market on a particular portfolio. Column C, Level 2, shown in Fig. 1 changes the benchmarks of a universe into components against which portfolio returns are regressed. The process is similar to the one used for swap rates. Table 3 shows the original benchmarks and the first four components, for this fixed income universe.

Table 3: Principal Component Eigenvectors for Benchmarks

Orthogonal Analysis of Benchmark Indices in the Universe	All Index Rise	G/C-down HY-up	Int/Mtg-down, G/C-up	GC/Mtg/HY-down, Credit-up
Barclays US Aggregate	0.2316	-0.0416	-0.3195	-0.0851
Citigroup US Broad Inv Grade	0.2310	-0.0464	-0.3186	-0.0885
Barclays US Govt/Credit	0.2742	-0.0576	-0.1710	0.0212
Barclays US Intermediate Govt/Credit	0.1681	0.0235	-0.3164	0.0610
Barclays US Long Govt/Credit	**0.7358**	-0.3993	0.4512	-0.2226
rclays US Mortgage Backed Securities	0.1420	-0.0274	-0.6413	-0.3422
BofA Govt/Corp 1-3 Yr	0.0300	0.0322	-0.1025	0.0843
BofA High Yield Master	0.2756	0.8849	0.1779	-0.3297
Barclays US Credit	0.3887	0.2189	-0.1007	0.8356

In place of the original nine indices that capture market returns against which a single strategy was to be managed one at a time, the methodology uses four components whose returns capture 99.8% of the total variability in these original indices. The way in which each portfolio relates to the components results in separate and time-varying benchmark proxies. Active returns above proxies of these components should be attributed to manager skill. Level 3 of Column C, in Fig. 1 measures the sensitivity of each portfolio to these four indices though linear regression. The intercept in this series of linear equations is restricted to zero, so that benchmark-return alpha is disallowed. For example, the estimated equations for two bond strategies, one constrained and unconstrained to an index are:

$$r_{constr}^{bench} = 0.1413 r_{allIndexRise} - 0.0170 r_{G/CdnHYup} - 0.6817 r_{Int/MtgdnG/Cup} - 0.3107 r_{GC/Mtg/HYdnCredup}$$
$$r_{unconstr}^{bench} = 0.1447 r_{allIndexRise} + 0.0055 r_{G/CdnHYup} - 0.63687 r_{Int/MtgdnG/Cup} - 0.1836 r_{GC/Mtg/HYdnCredup}$$

Benchmark proxies, against which these two particular strategies are managed, are illustrated by the two equations above. Portfolio excess (active) returns are similarly defined as actual, minus proxy returns, estimated as above. Incidentally, one may notice the difference in these two strategies in reference to the second component of indices, *G/C-down HY-up*.In Table 3,Barclays Long Government/Credit index has a negative weight (-0.3933); Bank of America High Yield Master has a strong positive weight (0.8849). Thus,*G/C-down HY-up*merely shiftsfund outperformance from Long Government/Credit into High-Yield exposure (coefficients -0.0170 and +0.0055). Unconstrained strategies are best-compared to High-Yield, illustrating the necessity of re-defining benchmarks to account for most market variability.Probability of outperformance rests on ranking of resulting IR in a universe.

This probability is estimated as a function of the sensitivity of portfolios to swap ratesin a Logit model. As shown in Fig. 1, Level 3 under Column A, portfolio returns are regressed against components of swap rates, to find the sensitivity of strategies to market forces. At the other end, in Level 3 of Column C, the portfolio coefficients to index components determine the returns of the most representative benchmark for each portfolio in the universe.

Subsequently excess returns, tracking error and the information ratio for each portfolio are estimated in Level 4 under Column B. The probability of outperformance is based on the density of the ranked information ratios of the portfolios in a fixed income universe. Each portfolio's information ratio is calculated as its average excess return, in a sample of the last 24 months, divided by the standard deviation (tracking error) in the same period. Given that the population excess (active) return is zero (on average, active management does not outperform, or underperform the representative market portfolio), the information ratio is a normal density *z*-value, the magnitude of which is estimated function by sensitivity of portfolios to components of swap curves, below.

$$\text{information ratio}: IR = \frac{(\overline{r_{portfolio}} - r_{benchmark}) - 0}{stdDev(\overline{r_{portfolio}} - r_{benchmark})}$$

$$r_{portfolio} = f(coefficientstoswapratecomponents)$$

The first of the two equations above shows that information ratio is similar in concept to a standard density *z*-value, associated with discrete normal probability $p(IR_{rank=k})$. The second equation states one variable of this *z*-value, $r_{portfolio}$ as a function of the estimated coefficients of portfolio returns against the components of swap rates, from Level 3 of Column A, in Fig. 1. The substitution of portfolio returns from the second equation into the first leads to the cumulative probability distribution of ranked information ratios, $P(IR_{rank=k})$. That is the accumulating sum of probabilities of ranked information ratios, up to the particular rank of the portfolio k. It is related to the k-coefficients to swap rate components through a non-linear (logistic) function $g(k)$, as follows:

$$P(IR_{rank=k}) = \sum_{-\infty}^{k} p(IR_j) = g(k - thportfoliocoefficientstoswapratecomponents)$$

Table 4: Estimation of Probability of Outperformance

Portfolio	Information Ratio:	P(IR)	Logit (P(IR))
1	58.61%	86.37%	185%
2	52.57%	78.48%	129%
3	14.61%	32.86%	-71%
4	85.06%	97.70%	375%
5	77.60%	95.98%	317%
6	34.02%	57.53%	30%
7	48.35%	75.04%	110%
8	42.14%	67.43%	73%
9	-23.31%	3.73%	-325%
10	82.17%	97.13%	352%
11	37.84%	59.97%	40%
12	59.67%	87.52%	195%
13	56.46%	83.07%	159%
14	62.71%	89.67%	216%
15	36.59%	59.40%	38%
16	40.56%	63.99%	57%
17	41.33%	65.57%	64%
18	-8.80%	12.05%	-199%
19	38.25%	60.98%	45%
20	-28.36%	2.44%	-369%
691	-13.40%	8.90%	-233%
692	122.45%	99.86%	655%
693	14.47%	32.86%	-71%
694	32.86%	55.09%	20%
695	86.01%	98.28%	404%
696	63.69%	90.53%	226%
697	59.74%	87.52%	195%

The sequence of the probability of outperformance estimation is (see Level 4, Columns A-B, Fig. 1):

1. Obtain the information ratio for each portfolio, from excess returns, shown in the 'information ratio' column of Table 4, below. This information ratio is based on excess returns, Level 4, Column B, Fig. 1.
2. Divide the range of information ratio into buckets. Here, the highest and lowest information ratio values were 130% and -60%, respectively. This range is split into 200 buckets, resulting in buckets of size of 1%.

3. Derive the probability histogram of the information ratio of portfolios, in reference to the buckets found above. Super-impose a normal density with the same mean and variance and test if histogram is normal.
4. Find the cumulative probability distribution of the information ratio of the portfolios, by summing values $p(IR_{rank=k})$ that are derived in step 3 above, up to the k-th bucket, for all the buckets, from -60% to 130%.
5. Cross-reference the cumulative probability, from the probability bins described above, back to portfolios as shown in the '$P(IR_{rank=k})$' column of Table 4 to arrive at the dependent variable of a logistic regression.

The last column in Table 4 is the logit transformation of cumulative probability of outperformance. It is required for linearizing *g(k-th portfolio coefficients to swap rate components)*. The logistic transformation of cumulative probabilities is further explained in Appendix I.Fig. 2 below, shows the estimated probability of outperformance (red squares) as a result of the logistic regression of actual probability (blue diamonds) against coefficients that measure the sensitivity of each portfolio to the principal components of swap rates. This model can be estimated across universes, such as Investment Grade, etc.The universe in which a portfolio belongs should not make a difference to its probability of outperformance once the largest portion of underlying market variables that drives returns is accounted for (data for this study obtained from a confidential source). But based on Fig. 2, a granular approach to probability of outperformance provides forecasts that lie closer to the actual probability.

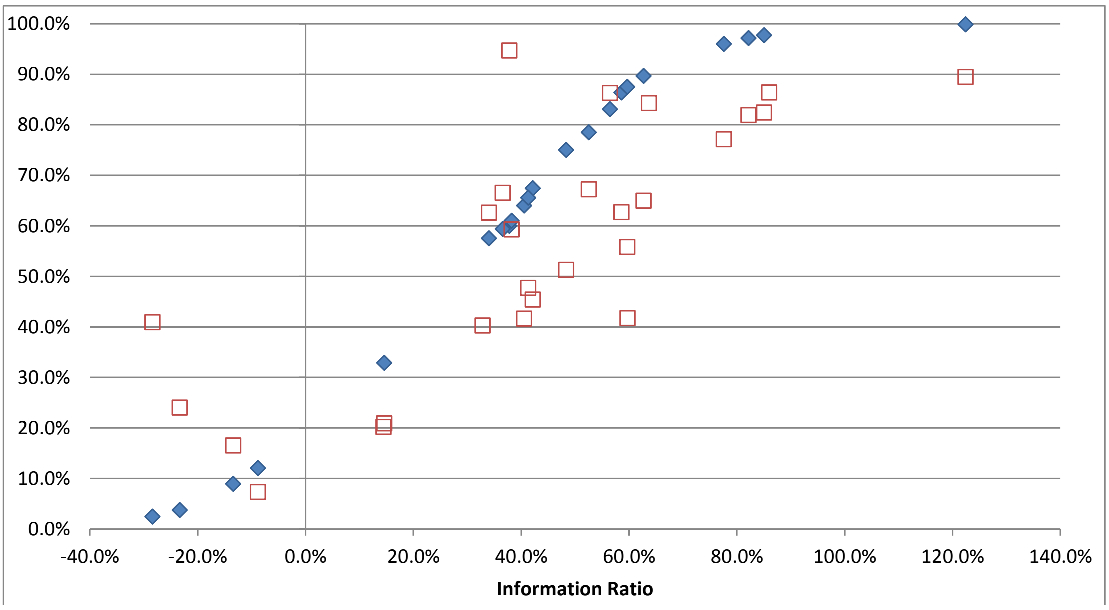

Figure 2: Logistic Regression of P(IR) on Portfolio Coefficients to Swap Rate Components

V.SEPARATION BETWEEN RATED STRATEGIES

Whereas the aim of the analysis so far was to forecast the probability of outperformance for a portfolio, the goal in this section is to categorize the characteristics of portfolios that are separated into recommended and non-recommended classifications. In industry practice, this separation is driven by human interactive processes of the 'expert' investment consultants with the portfolio management firms, and institutional investors. The concern for both institutional investors and regulators is that these recommendations by consultants fail to outperform in the long run. The issue examined in this analysis is different. Specifically, regulators and the investment community may be interested in ascertaining the contemporaneous, as opposed to the long-run effect of recommendations on outperformance gains or losses incurred by investors. The intent of this study is to propose a possible methodology that could aid in that particular goal. This section describes the*Z*-score for each strategy in any universe examined. Thediscriminant *Z*-score is a function of sensitivities of the portfolios to the underlying market forces used above. The degree to which the consultants' rating processes accurately separate strategies into recommended and non-recommended, is reflected in the manner that this *Z*-score creates separation. It exhibitsas little overlap of scores between the two categories as possible. This adequate separation between two classifications is initially unrelated to outperformance. The *Z*-score only separates strategies into categories.Strategies recommended correspond to a certain 'cocktail' of portfolio sensitivities to the swap rate components, different from the non-recommended category. Once a *Z*-score separates categories adequately, the comparison to probability of outperformance in the previous section becomes easy. The challenge is not to discern which of these two, outperformance probability, or the separating*Z*- score, is more precise. The hope is that the method offers hints toward uncovering the 'expert' practices where particular attention should be paid to, by regulators and hopefully, by the consultants themselves.

Discriminant analysis is largely suited to finding a relationship between a dichotomous dependent variable and a set of independent variables that separate that dichotomous variable. Applied to strategy recommendations, this analysis uses the same independent variables as outperformance to distinguish portfolios among two categories. The discriminant *Z*-score determines if a portfolio belongs to the recommended or the non-recommended category, purely based on sensitivity to components of swap rates. For example, if strategy *k*, in a universe *u*, has a sensitivity to the level *L* and slope *S*of swap rates (first two components) equal to $b_L(k)$ and $b_S(k)$, respectively, its *Z*-score, capital-*Z*(*k*), would beestimated, as in the first of the three equations below (please see Appendix II):

$$\lambda_L z_L(k) + \lambda_S z_S(k) = Z(k)$$

$$z_{L(k)} = \frac{b_L(k) - b_L(u)}{S_{b_L(u)}}$$

$$z_{S(k)} = \frac{b_S(k) - b_S(u)}{S_{b_S(u)}}$$

The last two equations, lower-case $z_{L(k)}$ and $z_{S(k)}$,merely standardize *b*( ) values into normally distributed*z*values. The$\lambda_{L(evel)}$and$\lambda_{S(lope)}$coefficients are determined such that recommended (non-recommended) strategies have a positive (negative) *Z*-score. Based on this relationship, 'expert' recommendation of an unrated strategy is found by substitution of its values for coefficients $b_L(k)$ and $b_S(k)$ into the above equations. For ease of exposition, only two of the components of swap rates, level-*L* and slope-*S*, are shown in the equations, and in the Appendix. The discriminant function scores across universes were estimated. It was realized that the coefficients differed greatly, among all universes. The discussion below pertains to universe *Long Duration*. The premise that recommended strategies reflect a higher probability of outperformance was examined through comparingthe logit-probabilities to the discriminant *Z*-scores. To test if the relation between these variables was positive, linear regression between these two was run (red line, in Fig. 3 below).Points that fall on the blue line are combinations of *Z*-scores and outperformance probabilities, for rated strategies. Most rated strategies in Long Duration fixed income are shown to follow probability of outperformance, closely. Several aspects of this diagram are worth noting:

i. Recommended(non-recommended) strategies have a positive (negative) *Z*-score.
ii. The positive slope between outperformance and *Z*-scores showsthe efficacy in ratings.
iii. It is easy to pinpoint an outlier that does not conform, and to find reasons behind the deviation.

Table 5 lists strategies that are recommended, and not recommended, in Long Duration. Column "Discriminant Function Score" is plotted against "Estimated Probability of Outperformance," in Fig. 3. Coefficients to swap rate components explain both the*Z*-score and the probability of outperformance. The outlier strategy with *Z*-score of -9.5844 (non-recommended) and outperformance probability of 76.4% (top quartile) has sensitivity to Persistent Quantitative Easing,of 2.2767 (highest overall). Assuming *Z*-scores are accurate, 'expert' consultants could have classified this one as recommended, instead. On the other hand, this strategy may simply 'front-run QE.'In this case, consultantsmay be injectingpersonal bias into investors' desirability of deriving alpha from possible front-running of central bank intervention. Whether such bias is clearly revealed in client-facing meetings, is arguable.

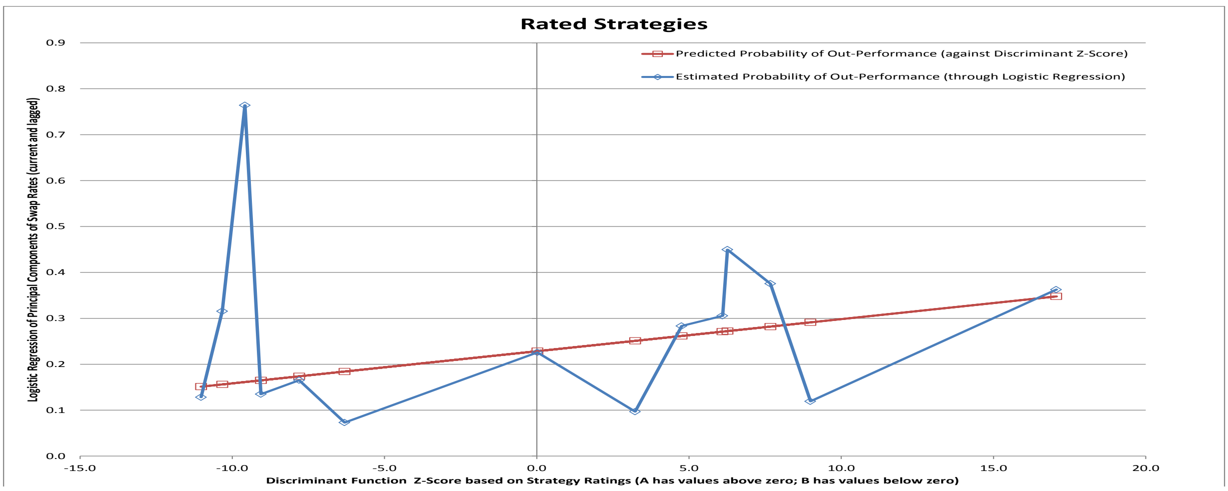

Figure 3: Discriminant *Z*-Score versus Probability of Outperformance

Table 5: Explanation of Relation between *Z*-score and Outperformance – Long Duration

Long Duration		Swap Rates Up for Quarter	Swap Rates Flat for Quarter	Current Down/LLagged Up	Persistent Quant. Easing	Discriminant Function Z-Score	Rating Average	P(IR)	Estimated Probability of Out-Performan
Portfolio 1	Recommended	1.0875	1.0790	-1.5468	0.6923	17.0636		47.1%	36.3%
Portfolio 2		1.5744	1.7562	0.5433	0.4468	8.9903		26.0%	11.9%
Portfolio 3		1.1044	0.9685	-1.2036	1.0518	7.6741		67.4%	37.6%
Portfolio 4		0.1160	0.1309	-0.5807	0.0435	6.2638		92.3%	45.0%
Portfolio 5		0.3863	0.3431	-0.7214	0.3197	6.1024		57.5%	30.6%
Portfolio 6		0.4064	0.3609	-0.8014	0.5045	4.7597		91.2%	28.3%
Portfolio 7		-0.7062	-0.7435	-0.3789	-0.6369	3.2323		6.0%	9.7%
Portfolio 8		0.1714	0.2036	0.1355	0.0860	0.0217	6.7635	24.1%	22.6%
Portfolio 9	Non-Recommended	-1.3559	-1.3636	0.8445	-1.2976	-6.3102		12.1%	7.3%
Portfolio 10		-0.7437	-0.7960	0.4671	-0.4214	-7.8012		8.9%	16.5%
Portfolio 11		-1.1326	-1.1705	0.4421	-0.6154	-9.0601		15.8%	13.5%
Portfolio 12 (outlier)		1.3887	1.3515	-0.4531	2.2767	-9.5844		72.2%	76.4%
Portfolio 13		-1.0406	-0.9681	0.7388	-0.5111	-10.3289		17.1%	31.5%
Portfolio 14		-1.2560	-1.1520	2.5147	-1.9387	-11.0233	-9.0180	2.4%	12.8%

A way to measure the relation of separation by discriminant function, to probability of outperformance,is linear regression of probability of outperformance against the discriminant function *Z*-score, reflectedin Fig. 4 below. Despite the small number of observations (13), therelation is statistically significant at the 10% levelafter taking the outlierout of the data. The 0.01 coefficient and the 0.08 *p*-value indicate that the two variables are positively related. The recommended strategies in this universe correspond to high probability of out-performance after the outlier is taken out. This kind of information can be gleaned from this analysis of portfolio outperformance against a discriminant function score based purely on contemporaneous rating. The methodology can be extended to other universes, and augmented to account for various other independent variables, data for which may be available.

SUMMARY OUTPUT

Regression Statistics	
Multiple R	50.54%
R Square	25.55%
Adjusted R Square	18.78%
Standard Error	11.07%
Observations	13

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	0.05	0.05	3.77	0.08
Residual	11	0.13	0.01		
Total	12	0.18			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*
Intercept	0.23	0.03	7.41	0.00	0.16
Discriminant Function Score	0.01	0.00	1.94	0.08	0.00

Figure 4: Summary Statistics of Relation between *Z*-score and Outperformance – Long Duration

VI.CONCLUSION

A methodology is proposed, which pairs probability of outperformance against recommendation for investment. Several standard, statistical techniques are employed. The methodology entails methods encountered in finance, such as principal component analysis, logistic regression, and the simplest, most-original of categorical analyses.The implementation of this methodology offers clues as to a relation between recommended strategies, and their probability of outperforming a proxy. The forces that drive performance are arranged in a parsimonious manner that captures most of swap curve variability. Portfolio sensitivity to these forces serves as independent variable in outperformance and in rating-separation. In the case of discrepancy between ratings and outperformance, reasons behind it are pointed out for the purpose of focusing 'expert' recommendations toward client-desired outcomes.The methodology is of interest, in the face of "Fiduciary Rule" regulation of the investment management industry.

VII.APPENDIX I: PRINCIPAL COMPONENTS, LOGISTIC REGRESSION

Generally, principal components of swap curves are linear combinations of these original variables, in a manner similar to the way portfolio returns are defined; that is, some linear combinations of asset returns. The special characteristic of principal components is that the eigenvectors are uncorrelated with each other. In addition, each component sequentially explains as much of the variation in the original variables as is possible. Thus, finding these new, hidden dimensions requires use of an optimization algorithm. Generally, if the original variables are represented by a matrix $\boldsymbol{X}$, and the square matrix with which they are multiplied in order to be changed into principal components is $\boldsymbol{P}$, the components are $\boldsymbol{X}$ times $\boldsymbol{P}$. Matrix $\boldsymbol{P}$ is called the matrix of eigenvectors, while$\boldsymbol{\Lambda}$ is the matrix of eigenvalues. We denote the covariance matrix of the original variables, as$\boldsymbol{\Omega}$. Covariance of the principal components is$\boldsymbol{P^T \Omega P}$, where superscript '*T*' denotes 'transposed.' The PCA method seeks to find vector $\boldsymbol{P}$ that makes the new covariance equal to a diagonal matrix, in which elements off the diagonal capture the covariance between components, while elements on the diagonal show the variance of components. The former are zero, while the latter are the largest possible, in descending order. Components capture the largest uncorrelated variation in any data. Research efforts can focus on how a dependent variable $\boldsymbol{Y}$, such as portfolio performance, responds to these hidden characteristics of the original, $\boldsymbol{X}$ variables. These responses are not apparent in the original data of swap rates, for example. The representation of underlying market forces by principal components ensures that most influences on the performance of strategies are described in the most parsimonious way [3].

The potential for assigning a score between 0 and 100 to the performance of a portfolio is a case in which the variable of interest (a*Z*-score) is close to binary, or follows a transition function (S-shaped curve). This function applies to the probability of outperformance described above, as a function of attributes of the portfolio (in this case, the sensitivities to principal components of swap rates). This binary-like score can thus be estimated as a function of independent variables, similar to a linear regression model. The difference is that a dependent variable takes values between 0 and 1 only, and transitions smoothly between these values. A linear regression model does not serve this purpose well, because its predicted values can range above 1, and/or below 0. A Logit model uses a nonlinear regression function that ensures the result lies in the interval [0,1]. The Logit model is based on the cumulative logistic probability of the information ratio, $P(IR)$, as a nonlinear function of x, below [4]:

$$P(IR) = \frac{1}{1+e^{-(\alpha+\beta x)}} \Rightarrow ln\left(\frac{P(IR)}{1-P(IR)}\right) = \alpha + \beta x$$

In the above relation, the cumulative probability of the information ratio $P(IR)$ is a nonlinear function $g(\ )$ of the variable x, which in this case is a vector of the sensitivity of a portfolio to the four components of swap rates, as discussed above. After the logistic transformation takes place, the model becomes linear and can be estimated through OLS, although maximum likelihood estimation is recommended for a number of econometric reasons.The purpose of developing this Logit model is to compare results against the discriminant function score, which is explained below. No claim is made that the logistic model captures the likelihood of outperformance 'better.' The method only points to potential sources of explanation, represented by variable x, in the logistic regression.

VIII.APPENDIX II: DISCRIMINANT ANALYSIS

In this analysis the simplest of discriminant analysis methods is used [5]. The method was originally proposed by Fisher (1936) [6]. It is based on a common covariance matrix between the categories 'recommended' and 'non-recommended' as far as this study is concerned. Since then, several augmentations have been proposed, and even programmed in modern statistical packages, such as *R*® [7]. An analogy of discriminant analysis to ordinary least squares (linear regression) may shed light into the nature of this method. Although both techniques are used as part of the overall here, the purpose of each one is different. While regression predicts the dependent variable, discriminant analysis is dependent-variable driven, instead.In linear regression, the dependent variable is assumed to be normally distributed. Here, independent (explanatory) variables would have values that are predetermined. In contrast, the discriminant analysis has a dependent variable with predetermined values that correspond to recommend, and not-recommend, categories.Linear regression predicts the values of a dependent variable with an estimated model of the independent variables. In comparison, discriminant analysis finds the optimal linear combination of independent variables, which minimizes the misclassification of strategies into two groups.Linear regression generates parameter estimates that have desirable properties, and invokes certain assumptions about the properties of these estimates. A discriminant function, on the other hand, is a method of classifying objects into groups, and of measuring the accuracy of classification. In finding a discriminant *Z*-score, the methodology is adapted to that in the original 1936 publication of R. AFisher. Strategiesrated by 'expert' consultants are separated into recommended and nonrecommendedgroups. The goal is to find $\lambda_{L(evel\,)}$and$\lambda_{S(lope\,)}$coefficients that optimize the separation between the rating groups. The optimized function of *b*( ) values is achieved through a standard maximization of the expression below. This problem of discrimination was initiated by Fisher (1936).

$$\Delta = \frac{\left\{[\lambda_L,\lambda_S]\begin{bmatrix} b_L(A) \\ b_S(A) \end{bmatrix} - [\lambda_L,\lambda_S]\begin{bmatrix} b_L(B) \\ b_S(B) \end{bmatrix}\right\}^2}{[\lambda_L,\lambda_S]\begin{bmatrix} s_L^2 & s_{L,S} \\ s_{S,L} & s_S^2 \end{bmatrix}\begin{bmatrix} \lambda_L \\ \lambda_S \end{bmatrix}} = \frac{\{\boldsymbol{\lambda d}\}^2}{\boldsymbol{\lambda}^T \boldsymbol{S\lambda}}$$

$$[\lambda_L,\lambda_S] = \begin{bmatrix} s_L^2 & s_{L,S} \\ s_{S,L} & s_S^2 \end{bmatrix}^{-1} \left\{\begin{bmatrix} b_L(A) \\ b_S(A) \end{bmatrix} - \begin{bmatrix} b_L(B) \\ b_S(B) \end{bmatrix}\right\}$$

The second equation is the optimal solution. It is the vector of coefficients in the *Z*-score function that separate strategies after substitution of the sensitivity of a portfolio to components of the swap curve. The vector$\boldsymbol{\lambda}$ is the result of setting the first derivatives of the ratio $\boldsymbol{\Delta}$ with respect to each of the coefficients, equal to zero. The resulting *Z*-score takes positive and negative values.The method rests on plotting probabilities of outperformance against *Z*-scores, and on gauging causes of misalignment between the two in a universe of fixed income portfolios. Lest the investor lost substantial portions of IR by listening to the 'expert' consultants'recommendations, whether the investor is better-off following simple outperformance metrics instead, is hard to tell, understandably. Looking at portfolio *b*( ) coefficients is important, in delving into causes of a misalignment of scores with outperformance.

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