

Value-at-Risk Implied in Black-Scholes Model to Calculate Option Prices

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ABSTRACT : Option is a contract of agreement that gives rights to the option holder to buy or sell a reference asset (for this matter is stock) at a certain price and at a certain agreed time. To buy an option required a number of premiums to be paid or commonly referred to as prices reasonable option. The valuation of fair price that is often used is by the Black-Scholes model which consists of assuming no dividend distribution, interest rates and volatility are considered constant, there are no taxes and transaction fees calculated, and so on. In this study Facebook stock option prices (FB) will be calculated using the usual Black-Scholes method and modified Black-Scholes by replacing the volatility with the greatest risk value to be calculated by the Value-at-Risk method. From the results of the study of the FB stock price obtained that the option price obtained by the usual Black-Scholes method are cheaper than Black-Scholes method using Value-at-Risk.

KEYWORDS:- Black-Scholes, Investment, Option, Option Prices, Value-at-Risk

I. INTRODUCTION

Investment is a commitment to a number of funds used with a goal to get a number of benefits in the future [14]. Of the many types of investments available, it can be said that stock investments are developing and are starting to be popular with many people in Indonesia.

Every investment has its own advantages and risks, and investment in the stock market is no exception, stock prices that can change very quickly from high to low often make investors anxious, so there is another alternative to investing a stock that is stock options. According to Abudy (2007) [1] stock options are derivatives of stock in which the investor has the right to buy or sell underlying assets, namely option to the seller of rights at a certain price and at a certain time. From the type of use rights, stock options are divided into two types, European type where investors only can exercise their rights right at maturity and the type of America where this option gives investors the freedom to exercise their rights during the beginning to the end of the agreement. The Black-Scholes model is a model for determining option prices that have been widely accepted and recognized by the financial community. This model was developed by Fisher Black and Myron Scholes in early 1973 (Black, 1973). This model has the form of a differential equation. So, we can determine the value of call options and put options numerically. In 2016 three lecturers from Padjadjaran University also examined the effect of interest rates on the calculation of stock option contracts using the Black-Scholes model. It is known that the greater the interest rate, the greater the value of option prices [10].

Based on this description, the writing of this paper will discuss the application of Value-at-Risk in the Black-Scholes model to calculate the price value of European-type stock options, which aims to determine the fair price of a traded option, which is expected to minimize the potential loss will be obtained by an investor, and can be a consideration for an investor when he wants to buy an option. The option price to be calculated is the option price from Facebook (FB) which uses stock price closing data from related companies, namely the Facebook company (FB).

II. THEORETICAL FRAMEWORK

2.1 Investment on Option Price

According to Sunariyah (2004) [13], investment is investment for one or more assets that are owned, and usually for a long time in the hope of getting a profit in the future. Investment transactions usually occur in a capital market, which is a market that is prepared to trade stocks, bonds, and other types of securities using the services of a securities broker.

Option is a contract of agreement that gives rights to the option holder to buy or sell a reference asset (for this matter is stock) at a certain price and at a certain agreed time [4]. The party who gets the right is called an option buyer or is also called an option holder, while the party selling the option and must be responsible for the decision of the option buyer when the option will be used is called the option writer. The time limit for the option to expire is called the due date (expiration date), and the price of the asset agreed upon by the writer and buyer is called the strike price or exercise price.

Options can be distinguished based on time of implementation [6]:

1. European option, which is an option that can be used (exercised) only on the due date.
2. American type options, namely options that can be used (exercised) before or on the due date.

Options can also be distinguished based on their functions [6]:

1. Option to buy (call option), the option that gives the right (but not the obligation) to the holder to buy certain assets at a certain price and a predetermined time.
2. Put option, which is the option that gives the holder the right (but not the obligation) to sell the specified asset at a specified price and at a specified time.

2.2 Option Price

According to Fabozzi and Markowitz (2002) [5], option prices are a reflection of the intrinsic value of the option and any additional amount of intrinsic value. Premium over the intrinsic value is called the time value or premium time.

Tabel 1. Intrinsic Value

Explanation	Intrinsic Value	
	Call Option	Put Option
If Stock $(S_T) >$ Strike Price (K)	$S_T - K$ (In The Money)	0 (Out the Money)
If Stock $(S_T) =$ Strike Price (K)	0 (At the Money)	0 (At the Money)
If Stock $(S_T) <$ Strike Price (K)	0 (Out the Money)	$K - S_T$ (In The Money)

2.3 Stock Price Movement

Stock price is a random variable whose movements are not known with certainty and can change at any time and the time interval of change is unknown, so it can be said that stock prices follow a continuous stochastic process. The continuous observation of this stock price movement turns out to resemble the movement of gas particles that move randomly (Brownian Motion).

2.3.1 Brownian Motion

A stochastic process W_t with $t \geq 0$ dikatakan proses gerak Brown jika [4]:

1. W_t is a continuous function and $W(0) = 0$
2. $\{W_t, t \geq 0\}$ has a free rise and a stationary rise
3. $W_t > 0$, W_t normal distribution with an average of 0 and variance t.

2.3.2 Brownian Motion with Drift

Brownian motion has an average value of zero, while stock prices in a certain period usually move with a certain growth rate which is illustrated by the drift factor (μ). If W is a Brownian Motion Raw process, to describe the rate of price growth, then the stochastic process $(X_t, t \geq 0)$ defined as follows [7]:

$$X_t = \mu t + W_t \tag{1}$$

said as Brownian Motion with drift.

2.3.3 Geometric Brownian Motion in Stock

In Brownian Motion with Drift and Noise scale still has weaknesses as a model of stock price changes because it still allows negative values. For this reason, there are variations of Brownian Motion that can be developed into Geometric Brownian Motion, so that the stock model can be defined with [7]:

$$S_t = S_0 \exp(X_t) = S_0 \exp(\mu t + \sigma W_t) \tag{2}$$

Called Geometric Brownian Motion with parameters μ and σ^2 .

2.3.4 Lemma Ito

In the previous discussion it has been said that the share price follows the Wiener Process which has a drift (μ) constant and constant noise (σ), but the model fails or is not suitable for aspects of the price of the stock itself, because the price of the stock option is a function based on stock price and time. If X is the share price, and the stock price changes following the Ito process as follows [7]:

$$dx = a(x, t)dt + b(x, y)dW \tag{3}$$

where dW is Brownian Motion and a and b are functions of x and t , Variable x has a drift factor of a and variance b^2 , while $f(x, t)$ is the option value at time t , the option price is [15]:

$$dF = \left\{ \frac{\partial F}{\partial x} a(x, t) + \frac{\partial F}{\partial t} + \frac{1}{2} \frac{\partial^2 F}{\partial x^2} b^2(x, t) \right\} dt + \frac{\partial F}{\partial x} b(x, t) dW \tag{4}$$

with dW is the same Brownian Motion in equation (3) and F follows the Ito process in equation (4) with a drift factor of:

$$a \frac{\partial F}{\partial x} + \frac{\partial F}{\partial t} + \frac{1}{2} \frac{\partial^2 F}{\partial x^2} b^2$$

and the variance of:

$$\frac{\partial F}{\partial x^2} b^2.$$

So if an S share price is declared to follow The Geometric Brownian Motion(2) which is differentiated into:

$$dS = \mu S dt + \sigma S dW_t \quad (5)$$

with μ and σ constant. From the Ito lemma equation (4) and assume that $G = \ln S$, then the stock price has a function:

$$dG = \left(\mu S \frac{\partial G}{\partial S} + \frac{\partial G}{\partial t} + \frac{1}{2} \frac{\partial^2 G}{\partial S^2} \sigma^2 S^2 \right) + \frac{\partial G}{\partial S} \sigma S dz \quad (6)$$

which is obtained from equation (5) which is integrated from 0 to time T , because
 $G = \ln S$

so,

$$\frac{\partial G}{\partial S} = \frac{1}{S}, \frac{\partial^2 G}{\partial S^2} = -\frac{1}{S^2}, \frac{\partial G}{\partial t} = 0$$

from equation (6), the stock price process with function G is

$$dG = \left(\mu - \frac{\sigma^2}{2} \right) dt + \sigma dz \quad (7)$$

by substituting equation (7) and using the Brownian Motion geometric equation in equation (2) then the stock price S is expressed by:

$$S_t = S_0 \exp \left[\left(\mu - \frac{1}{2} \sigma^2 \right) t + \sigma W_t \right] \quad (8)$$

with $W_t \sim N(\mu, \sigma^2)$, where W_t is Brownian Motion.

2.4 Kolmogorov-Smirnov

X_1, X_2, \dots, X_n a random sample of size n from a population with an unknown $F(x)$ distribution function (Tse, 2009).

- Hypothesis Formulation:

$$H_0: F(x) = F_0(x)$$

$$H_1: F(x) \neq F_0(x)$$

- Statistics Test:

$$D_n = \sup |F_n(x) - F_0(x)| \quad (9)$$

with:

$F_n(x)$: Empirical distribution function

F_0 : Distribution function

Reject H_0 if $D_n > d_{n,\alpha}$

2.5 Volatility

Volatility is a very important thing in the financial field. This observation of volatility is used to describe changes in returns in the financial sector by using a standard measure of deviations over a period of time. The word volatility has a meaning of fast / easy to change in English terminology. So volatility itself is a standard deviation that continues to change over time.

Volatility per year can be calculated by the following equation [7]:

$$\hat{\sigma} = \sqrt{S^2} = \sqrt{\frac{\sum_{t=1}^n (R_t - R)^2}{n-1}} \quad (10)$$

with,

n : a lot of observations

R_t : rate of return at the time of t / Log return

R : The average log return

to get the values of $\hat{\sigma}$ and look for the following equation:

$$R_t = \ln \frac{s(t)}{s(t-1)} \quad (11)$$

with,

$s(t)$: stock price at the time of t

$s(t-1)$: stock price at the time of $t-1$

2.6 Value-at-Risk with the Normal Distribution

Approach which is usually abbreviated as VaR is defined as the maximum loss that might occur for a probability that is defined as a confidence coefficient (confidence coefficient) over a certain period of time or period. Meanwhile, Ruppert (2004) [11], VaR is defined as a market risk limit that can be estimated so that the loss during a certain horizon time is smaller than the loss limit, with the chance of occurrence of a certain

confidence coefficient. VaR estimation with the normal distribution approach is a parametric estimation of VaR, meaning that it is a way to estimate VaR estimated using parameters. VaR estimates for stocks with 100% confidence coefficient $(1-\alpha)$ are [7]:

$$VaR(\alpha) = -W(\hat{\mu} + \Phi^{-1}(\alpha)\hat{\sigma}) \quad (12)$$

where W is the initial investment, $\hat{\mu}$ the mean of stock returns, $\hat{\sigma}$ standard deviation of stock returns, and Φ^{-1} shows the inverse of α or the inverse of the normal standard value Z which can be seen in the normal distribution table. The minus sign in the above equation shows that VaR is an estimate of loss.

2.7 Black-Scholes

One of the basic theories in the world of finance is arbitrage pricing theory. This theory is based on a law where two same assets cannot be sold at different prices. If there is a price difference between the same two assets, an arbitration process will occur. Investors will sell more expensive assets and buy other assets that are cheaper. With this process a profit is obtained that is equal to the difference in the price of the two assets without having to bear the risk. In the end this process equalizes the prices of the two assets. People who arbitrage are called arbitrageurs, those who pay attention to market conditions first, then try to find a risk-free situation to gain profits by comparing two different markets.

Although the Black-Scholes model is intended for the valuation of call options, this model is also used to calculate the value of a put option using a balance between put-call options, the relationship is stated as follows:

$$C = S_0N(d_1) - Xe^{-rT}N(d_2) \quad (13)$$

Although the Black-Scholes model is intended for the valuation of call options, this model is also used to calculate the value of a sell option (put option) by using a balance between put-call options, the relationship is stated as follows:

$$P - C = Xe^{-rT} - S_0 \quad (14)$$

if the buy option price is known, then the selling option price can be calculated at the same exercise price, maturity period and benchmark stock. From equation (12) another form can be written, namely:

$$P = C + Xe^{-rT} - S_0 \quad (15)$$

substitution equation (12) to equation (14),

$$\begin{aligned} P &= S_0N(d_1) - Xe^{-rT}N(d_2) + Xe^{-rT} - S_0 \\ &= S_0[N(d_1) - 1] + Xe^{-rT}[1 - N(d_2)] \\ &= -S_0[1 - N(d_1)] + Xe^{-rT}[1 - N(d_2)] \\ &= -S_0[N(-d_1)] + Xe^{-rT}[N(-d_2)] \end{aligned}$$

so setting the selling option price with the Black-Scholes equation is:

$$P = -S_0[N(-d_1)] + Xe^{-rT}[N(-d_2)] \quad (16)$$

with,

P : selling option prices

C : buy option price

Xe^{-rt} : the present value of the exercise price

S_0 : stock prices

$N(d_1)$: normal distributive cumulative density function of d_1

$N(d_2)$: normal distributive cumulative density function of d_2

and,

$$d_1 = \frac{\ln\left(\frac{S_0}{X}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \quad (17)$$

$$d_2 = \frac{\ln\left(\frac{S_0}{X}\right) + \left(r - \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T} \quad (18)$$

where,

S_0 : initial stock price

X : strike price

r : risk free interest rates

σ : volatility of return

T : time to maturity

III. RESEARCH OBJECTS AND METHODS

The research methodology used is the study of literature conducted by collecting literature from previous research in accordance with the studies discussed in this study. The sources used come from journals, articles and recommended books.

The object of research in this thesis is secondary data obtained from Yahoo Finance. The observation unit used is the daily closing price of shares of Facebook shares (FB) for the period of 20 May 2019 to 6 December 2019. Calculation of option prices uses the Black-Scholes method and the Black-Scholes method in which volatility is replaced by the VaR value.

The assumptions used in the model results of this study are as follows:

1. Movement of stock values follows random motion
2. Valuation only at maturity (European type)
3. The value of volatility in stock price movements is constant
4. The risk-free interest rate is fixed and constant, with the interest rate set at 1.54% from the website <https://www.treasury.gov/resourcecenter/datachartcenter/interestrates/Pages/TextView.aspx?Data=billrate>.
5. The asset model is normally distributed and dividends are not taken into account
6. The due time is 41 days.

3.1 ResearchSteps

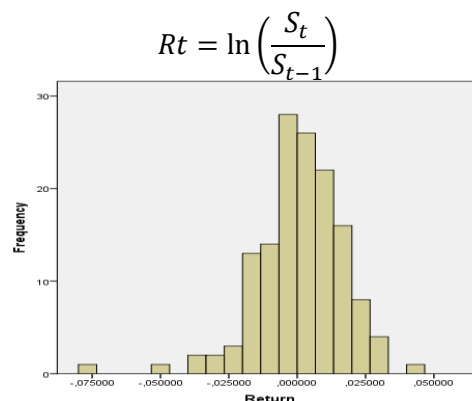
This research was conducted in several steps of analysis, which are as follows:

1. Determine the shares to be counted.
2. Calculate stock returns with equation (11)
3. Test the normality of stock return data by using the Kolmogorov Smirnov normality test from equation (9)
4. Calculate the value of volatility determined from the estimated standard deviation of stock price return data from equation (10).
5. Calculate theoretical prices of sell options and buy options through the Black-Scholes equation (13) and (16).
6. Calculating the VaR value using the normal distribution approach obtained through equation (12).
7. Calculate the theoretical price of sell options and buy options through equations (13) and (16) but change the value of volatility with the value obtained from the VaR calculation results of the normal distribution approach.
8. Comparing the two calculation results obtained in numbers 5 and 7.

IV. FIGURES AND TABLES

4.1 Return

The data used in this study are secondary data, namely historical data from the daily closing price of stock of Facebook (FB), from 20 May 2019 to 6 December 2019, obtained from Yahoo Finance accessed on Monday 16 December 2019 at 20.37 WIB Option prices are calculated for European call and put options. Then the next step is to calculate the closing stock return as follows:



Picture 1. Return of Closing Price

4.2 Normality Test

Normality of stock data returns is a condition of using the Black-Scholes method. The hypothesis in the normality test of stock returns is as follows:

H_0 :Return normal stock data

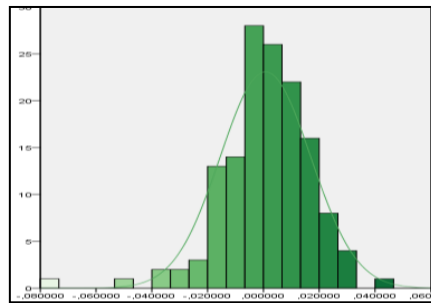
H_1 :Return of stock data is not normally distributed

The test statistic used in the Kolmogorof-Smirnov test is that H_0 is rejected if $\text{Asymp.sig} < \alpha$

Tabel2.Normality Test of ReturnFB

n	141
Mean	0,000710684
Std. Deviation	0,012130541
Asymp.sig	0,076

Normality test results performed with SPSS 22 Software on FB stock returns with Asymp.sig> Alpha, with Asymp.sig sequential values of 0.76 and selected Alpha values of 0.05 so it can be concluded that stock return data is normally distributed.



Picture 2. Normality of Return

4.3 Volatility

Based on FB stock data for the period 20 May 2019 - 6 December 2019, obtained volatility from stock data return of 0.0161305. this value can calculate with equation (10) This means that the average deviation of stock return movements from the average is 0.0161305.

4.4 Value-at-Risk Normal Distribution

The stages of VaR estimation using the normal distribution approach on FB shares are as follows:

1. obtained the mean and standard deviation for FB returns are 0,0007107 and 0.01624250. Where the value of the standard deviation is equal to the value of volatility.
2. For the 95% confidence coefficient obtained $\alpha = 0.05$ so that $\phi^{-1}(0.05) = -1,645$. Values $\phi^{-1}(0.05)$ can be seen in the normal distribution table, a negative sign indicates that the opportunity is under the left curve.
3. Assuming an initial investment (W) of 1 unit and based on equation (12) VaR is obtained for FB shares:

$$VaR(0,05) = -1[0,0007107 + (-1,645 \times 0,01624250)] = 0,02600823.$$

Which means that the biggest possibility for someone to lose while investing is 2.6%.

4.5 Option Price with Black-Scholes

In determining the price of stock options using the Black-Scholes method the first step that must be sought is to calculate the values of d_1 and d_2 . Using equations (2.44) and (2.45),

It is known that the volatility value of the FB stock price is 0.0162525, and the value of the volatility using VaR is 0.0260082. interestrates / Pages / TextViewasp? data = billrates and the expiration date of the stock options until 17 January 2020 (the age of the 41 day stock option contract). The choice of execution price is determined based on the available contract. If the result of the calculation is greater than the value of the options offered, the action that investors should take is to buy stock options. Conversely, if the calculation results are smaller than the value of the options offered, investors are advised not to buy. The value of the buy and sell options of FB shares can be calculated with equations (13) and (16). The calculation results for each Strike Price can be seen in Table 3 and Table 4.

Table 3. Stock Call Option Price

Strike Price	Real Option	Call Option BS	Advice	Call Option BS with VaR	Advice
10	190,6	172,738	No	172,738	No
20	151,25	162,755	Buy	162,755	Buy
30	153,43	152,773	No	152,773	No
40	109,15	142,79	Buy	142,79	Buy
50	142	132,808	No	132,808	No
60	132,2	122,825	No	122,825	No
70	120	112,843	No	112,843	No
80	111,5	102,86	No	102,86	No

90	99,15	92,8777	No	92,8777	No
100	100	82,8952	No	82,8952	No
110	90,92	72,9128	No	72,9128	No
120	81,39	62,9303	No	62,9303	No
130	73	52,9478	No	52,9478	No
140	54,85	42,9653	No	42,9653	No
150	44,93	32,9829	No	32,9829	No
160	35,45	23,0004	No	23,0004	No
170	25,07	13,0179	No	13,0179	No
180	14,99	3,03543	No	3,05262	No
190	5,3	3,20E-21	No	3,70E-06	No
200	0,52	1,00E-105	Buy	4,70E-25	Buy

Table 4. Stock Put Option Price

Strike Price	Real Option	Put Option BS	Advice	Put Option BS with VaR	Advice
10	0,04	0	No	0	No
20	0,02	0	No	0	No
30	0,1	0	No	0	No
40	0,02	0	No	0	No
50	0,01	0	No	0	No
60	0,03	0	No	0	No
70	0,01	0	No	0	No
80	0,04	0	No	0	No
90	0,01	0	No	0	No
100	0,01	0	No	0	No
110	0,01	0	No	0	No
120	0,02	0	No	0	No
130	0,02	0	No	0	No
140	0,01	0	No	2,00E-206	Buy
150	0,02	0	No	2,00E-115	Buy
160	0,03	8,00E-239	Buy	2,40E-54	Buy
170	0,07	1,40E-74	Buy	3,30E-18	Buy
180	0,22	3,50E-06	Buy	0,01719	No
190	1,14	6,94705	Buy	6,94706	Buy
200	6,4	16,9295	Buy	16,9295	Buy

4.6 Comparison of Value Option Price

In comparing the value of stock options using the usual Black-Scholes method with Black-Scholes using Value-at-Risk, the MSE value becomes a benchmark in determining the most appropriate model by looking at the smallest MSE value.

Table 5. MSE Call Option

X	Call Option	Call Option BS	Call Option BS with VaR	SE BS	SE BS with VaR
10	190,6	172,7375245	172,7375245	17,8625	17,86248
20	151,25	162,755048	162,755048	11,505	11,50505
30	153,43	152,7725716	152,7725716	0,65743	0,657428
40	109,15	142,7900951	142,7900951	33,6401	33,6401
50	142	132,8076186	132,8076186	9,19238	9,192381
60	132,2	122,8251421	122,8251421	9,37486	9,374858
70	120	112,8426656	112,8426656	7,15733	7,157334
80	111,5	102,8601891	102,8601891	8,63981	8,639811
90	99,15	92,87771266	92,87771266	6,27229	6,272287
100	100	82,89523617	82,89523617	17,1048	17,10476
110	90,92	72,91275969	72,91275969	18,0072	18,00724

120	81,39	62,93028321	62,93028321	18,4597	18,45972
130	73	52,94780672	52,94780672	20,0522	20,05219
140	54,85	42,96533024	42,96533024	11,8847	11,88467
150	44,93	32,98285376	32,98285376	11,9471	11,94715
160	35,45	23,00037728	23,00037728	12,4496	12,44962
170	25,07	13,01790079	13,01790079	12,0521	12,0521
180	14,99	3,035427821	3,052615936	11,9546	11,93738
190	5,3	3,20842E-21	3,72078E-06	5,3	5,299996
200	0,52	1,2221E-105	4,71614E-25	0,52	0,52
MSE				201,8154	201,7949

Table 6. MSE Put Option

X	Put Option	Put Option BS	Put Option BS with VaR	SE BS	SE BS with VaR
10	0,04	0	0	0,04	0,04
20	0,02	0	0	0,02	0,02
30	0,1	0	0	0,1	0,1
40	0,02	0	0	0,02	0,02
50	0,01	0	0	0,01	0,01
60	0,03	0	0	0,03	0,03
70	0,01	0	0	0,01	0,01
80	0,04	0	0	0,04	0,04
90	0,01	0	0	0,01	0,01
100	0,01	0	0	0,01	0,01
110	0,01	0	0	0,01	0,01
120	0,02	0	0	0,02	0,02
130	0,02	0	0	0,02	0,02
140	0,01	0	1,6782E-206	0,01	0,01
150	0,02	0	2,2003E-115	0,02	0,02
160	0,03	7,7474E-239	2,41306E-54	0,03	0,03
170	0,07	1,44386E-74	3,34054E-18	0,07	0,07
180	0,22	3,51014E-06	0,017191626	0,22	0,20281
190	1,14	6,947052172	6,947055893	5,80705	5,80706
200	6,4	16,92952866	16,92952866	10,5295	10,5295
MSE				17,0266	17,0094

V.CONCLUSION

From the results of calculations that have been done, it is found that the value with risk calculated by VaR gives the price of call and put options higher than the risk from the results of the calculation of ordinary variance. This is due to the higher level of risk, the calculation on the Black-Scholes model will result higher option prices. Value of FB stock option price is closer with the fair price value from the calculation of the Black Scholes model using the risk calculated with VaR, rather than ordinary. it is seen from a comparison of MSE values.

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