American Journal of Humanities and Social Sciences Research (AJHSSR)
e-ISSN : 2378-703X
Volume-07, Issue-09, pp-109-114
www.ajhssr.com
Research Paper Open Access

# About the Mathematical Philosophy of Gaston Bachelard

Jacques CHATUÉ[1], Yves Romeo NOUMBO ZAMBOU[2]
[1](Department of Philosophy, Psychology, Sociology, Professor- University of Dschang, Cameroon)
[2](Department of Philosophy, Psychology, Sociology, Doctorate Student- University of Dschang, Cameroon)

**ABSTRACT**: How to characterize the mathematical philosophy of Gaston Bachelard? This question cannot receive a clear and quick answer. It is practically impossible in any case to study the epistemology of Gaston Bachelard without being marked by the decisive importance conferred on mathematics. Moreover, it would be more accurate to speak of an omnipresence of mathematics which leads to a quite singular mathematical philosophy. Indeed, by closely scrutinizing all the ideas that structure the mathematical thought of Gaston Bachelard, and by articulating them, we realize that there is in this author a completely new mathematical philosophy, but sometimes unnoticed. Bachelard claims his philosophy of mathematics, beyond rationalist qualifications, as being a “constructed realism” and “metaphorism”. This reflection consists of an explanatory statement of this mathematical philosophy as it comes under the little-known denomination of mathematical realism of a new kind.

***KEYWORDS***: *Epistemology, realism, mathematics, rationalism.*

***RÉSUMÉ*** :*Comment caractériser la philosophie mathématique de Gaston Bachelard ? Cette question ne peut recevoir une réponse claire et rapide. Il est pratiquement impossible en tout cas d’étudier l’épistémologie de Gaston Bachelard sans être marqué par l’importance décisive conférée aux mathématiques. Il serait d’ailleurs plus exact de parler d’une omniprésence des mathématiques et qui débouche sur une philosophie mathématique tout à fait singulière. En effet, en scrutant de près l’ensemble des idées qui structurent la pensée mathématique de Gaston Bachelard, et en les articulant, on se rend à l’évidence qu’il y a chez cet auteur une philosophie mathématique tout à fait inédite, mais parfois inaperçue. Sa philosophie des mathématiques, Bachelard la revendique, au-delà des qualifications d’ordre rationalistes, comme étant un « réalisme construit » et « métaphorique ». La présente réflexion consiste en un exposé explicatif de cette philosophie mathématique en tant qu’elle se décline sous la dénomination peu connue de réalisme mathématique d’un genre nouveau.*

***Mots clés*** : *Épistémologie, réalisme, mathématiques, rationalisme.*

## I. INTRODUCTION

The epistemological thought of Gaston Bachelard takes shape in contact with the theory of relativity and quantum mechanics. These two major events in the history of science will induce a new scientific spirit which obliges, on the one hand, to approach science from a discontinuist angle, and on the other hand, to analyze how mathematical thought constitutes the axis of research and scientific work. It is the mathematical structures that preside over scientific heuristics. Mathematics has three main functions for Bachelard: 1) a heuristic function: mathematics is the engine of scientific research, it constitutes the axis of scientific discovery. 2) a demarcation or even discriminatory function: mathematics makes it possible to distinguish science from all other forms of non-scientific or pseudo-scientific knowledge. 3) an ontological function: mathematics makes it possible to penetrate the real, better, to recreate the real. It is relative to this last function that he affirms: "In the beginning is the Relationship, this is why mathematics reigns over the real" [1]. There is therefore in our author a mathematical realism whose contours it is necessary to scrutinize and specify. Bachelard obviously evokes several realisms, which he distinguishes and ranks. It is first of all “naive realism”, which projects onto all reality the model of immediate realities. Then “philosophical realism”, which posits the fundamental irrationality of reality. These two realisms have nothing scientific about them. They are different from the “scientific realism” of modern physics and chemistry, which is first a mathematically "instructed" realism, then a “scientific technical realism” as the instrumental realization of mathematical idealizations [2]. How, then, can we grasp the full singularity of this epistemological realism of mathematical essence? In other words, what are its characteristics?

---

[1]Bachelard, Gaston“Noumenon and microphysics”,in *Studies* , Paris, Vrin, 1970, p.19.
[2]Gaston Bachelard, *Applied Rationalism* (1949), Paris, PUF, 1975, p.137.

## II. OF THE EPISTEMOLOGICAL STATUS OF MATHEMATICS

The constructed and metaphorical mathematical realism of Bachelard, who acts as a mathematical philosopher, is elaborated polemically against the “ontological realism” of the Platonic or Aristotelian type, against the deductive mathematicism and the reductivist realism of Descartes and Meyerson, against pure formalism, axiomism and logicism. This mathematical philosophy is distinguished by the epistemological status conferred on mathematics. For Bachelard, science is a work of application of mathematical thought. Hence the concept of “applied rationalism”.

Science before Galileo is a meditation on qualities, characterized by a clear cut between mathematics and the sciences of reality. It is a science based on a split between the rational and the empirical, the abstract and the concrete, form and content. With the constitution, by the ancient Greeks, of a theoretical and rational mathematics, clearly detached from the empirical practices of calculation and surveying, knowledge was divided and divided into two very distinct levels: at the higher level, a purely intellectual, rational which, by giving us access to a world of eternal essences, of incorruptible realities, truly gives us science. At the lower level, it is sensible knowledge which, not exceeding the level of opinion, rivets the spirit on matter and phenomena in perpetual becoming. Thus, in spite of some remarkable exceptions like the work of Archimedes for the mechanics of the earth, mathematics was kept at a good distance from the studies of nature. This is how Greek physics, subservient to the diversity of sensory data, will remain a physics of qualities, far from preoccupations with quantities. It was only in modern times, under the impetus of Galileo, that physics was mathematized. Mathematics thus opened up to other areas of knowledge, notably physics. It is probably at this moment in history that we could situate the beginnings of an openness of mathematics to interdisciplinary, with regard to its strong coupling with physics.

In *Essay on approximate knowledge* , book which sets up his singularly constructivist mathematical philosophy, Bachelard starts from nuanced reflections on the Platonism to which mathematical forms invite, elaborating on what he calls “the progressive character of the metaphorical existence of mathematical being” [3]. The mathematical realism that he is then willing to admit stems not from Platonic realism where mathematical beings flourish in the autonomous existence of a stable eternity[4], but from a “constructive ontology”. "This constructive ontology, affirms our author, is never at its end since it corresponds rather to an action than to a discovery" [5], more precisely: which stems from a process of progressive reification where "the nature of the given objects gives way to the rules that govern them” and where mathematical beings have no intrinsic reality, their existence being relative to their domain of definition”. [6]For Bachelard, mathematical beings do not have an ontological existence in an ideal reality or an “intelligible world" in the manner of Plato, any more than in the sensible world, such as it is given to our usual experience, as well as conceives it Aristotle.

Plato did not just inaugurate the dualism of soul and body. He also developed a whole radical dualism between mathematical knowledge and empirical knowledge, his thesis being that mathematical beings, insofar as they are intelligible essences, basically have no relationship with the universe of materiality or sensible things of the world. This will be the beginning of a long intellectual tradition making mathematics an intelligible science which does not have to sully its purity by applying itself to the world of purely corruptible material phenomena. This explains why the word “Platonism” has become customary today to qualify this realism of mathematical essences which structures the thought of a good number of contemporary mathematicians for whom the relationship of mathematics to the other sciences is essentially a relationship *of application*, and not of *constitution* according to an Althusserian terminology. The fact is that Plato understood that any application of intelligible forms, including mathematics, to the world of materiality is a mark of decay. This opinion will not be shared by Bachelard, who believes on the contrary that for scientific rationalism, “ application is not mutilation; scientific action guided by mathematical rationalism is not a transaction on principles […] For scientific rationalism, application is not a defeat, a compromise” [7].

Against the deductive mathematicism of Descartes and against the reductivist realism of Meyerson [8], Bachelard proposes the inductive approach of mathematical noumena. His criticism of the deductive ideal leads him to posit that science proceeds from an essentially inductive approach, always opening up to novelty, to the unpublished. Moreover, noumenal induction suggests the passage from the mathematically thinkable to the physically possible. One cannot therefore logically reduce mathematics to a descriptive or reductive function. Mathematics has a creative power and the reality of phenomena is only inferred from the variables of the equations: "Bachelardian mathematical thought does not follow an identifying approach, reducing the

---

[3] Gaston Bachelard, *Essay on approximate knowledge* (1928), Paris, Vrin, 1973, p. 186.
[4]Cf. Plato, *The Republic, Paris* , trans. Robert Baccou, Garnier-Flammarion, 1966.
[5]Same.
[6]*Ibid*., pp.187-188.
[7]Gaston Bachelard, *The philosophy of no*, (1940), Paris, PUF, Quadrige, 1981,pp.6-7.
[8]SeeEmile Meyerson, *Identity and Reality*, Paris, Alcan, 1908.

particularities and diversities of cases to a minimum of similar and common characteristics. For him, mathematical thought is inventive thought [9].

Bachelard's mathematical thought also takes shape against mathematical formalism. The fault of formalism is to remain closed to any attempt at application. Mathematical activity does not flourish in its constitution as a closed formal system, kept at a distance from reality. For Bachelard, formalism is mathematics running on empty, seeing in application a form of mutilation:

> "If one condemns mathematical realism too soon, it is because one is seduced by the magnificent extension of formal epistemology, that is to say by a kind of empty functioning of mathematical notions. But if we do not unduly disregard the psychology of the mathematician, we soon realize that there is in mathematical activity more than a formal organization of schemes and that every pure idea is doubled by a psychological application, of an example that serves as reality. »[10]

Logicism is not spared from the Bachelardian critique. What characterizes logicism in fact, and which Bachelard decries, is not only the fact of founding mathematics in logic, but above all, the subjection of scientific reasoning to the respect of logical constraints which only apply to a stabilized formal structure. There is here a sort of mechanization of scientific reasoning, a mechanization that prevents us from apprehending mathematics as a "thought", a thought that fully masters its language:

> "We must break with this cliché dear to skeptical philosophers who only want to see mathematics as a *language* . On the contrary, mathematics is a *thought* , a thought sure of its language. The physicist thinks the experiment with this mathematical thought.[11]

We cannot therefore reduce mathematics "to the status of simple language which would express, in its own way, facts of observation". [12]Mathematics is not intended to summarize experience, it helps to think about it, to organize it.

## III. CHARACTERIZATION OF THE MATHEMATICAL PHILOSOPHY OF GASTON BACHELARD

It should be noted that the epistemological position of Gaston Bachelard is declined as a "scientific realism"[13], "a realism made of realized reason" which attributes to mathematics an ontological scope. Indeed, he posits the "scientific reality" [14]as emerging from a dual work of mathematical construction and technical production. The "scientific reality" is essentially a mathematical and technical "realization". The index of the real is the rationality of the mathematically constructed and technically produced phenomenon. This "scientific realism", which is developed at a distance from "vulgar realism" and "philosophical realism", animates physics and chemistry, in that it is "worked on", "rectified", "instructed" by mathematics. But still, physics and chemistry are of the order of a "realism of scientific technique" [15], that is to say a technical realization of mathematical idealizations, in particular algebraic. Bachelard thus grants an ontological existence to mathematical idealities, not in the sense that mathematics would exist in itself, as essences in a world separated from the physical universe, but rather in the sense that it is mathematics that allows to penetrate the real, and participate upstream in the work allowing to invent "from scratch" [16]the phenomena. In 1932, Bachelard explained that, contrary to what was thought in the 19th century, namely that science is "real through its objects", today, "It is now the objects which are represented by metaphors; It is their [mathematical] organization that appears to be reality"[17]. The physical object, "it is the mathematical formula which will give it a shape"[18]. Consequently, reality "has a mathematical meaning before having a phenomenal meaning"[19]. In *The Philosophy of No*, Bachelard states:

> "Reality turns into mathematical realism, then mathematical realism dissolves into a kind of realism of quantum probabilities.The philosopher who follows the discipline of

[9]Jacques Chatué et Al., *General introduction to epistemology*, Dschang University Press, 2020, p. 40.
[10] Gaston Bachelard, *The new scientific spirit* (1934),Paris, PUF,1984, p.8.
[11]Gaston Bachelard, *The rationalist activity of contemporary physics*, Paris, PUF, 1951, p.42.
[12] Dominique Lecourt, *The philosophy of science* , Paris, PUF, "Que sais-je?", 2001, p.102.
[13]See *Applied Rationalism* , Paris, Quadrige/PUF, 1949, p. 8.
[14] Gaston. Bachelard, *The New Scientific Spirit*, *op. cit.*, p.9.
[15]*Ibid.*, p. 137.
[16]See "Noumene and microphysics" in *Etudes* , Paris, Vrin, 1970, p.19.
[17]*Ibid.*, p.13.
[18]*Ibid.*, p.15.
[19]*Ibid.*, p.17.

> quanta – the schola quantorum – accepts to think all of reality in its mathematical organization"[20].

The characterization of Bachelard's thought as "scientific realism" is however neither sufficient nor primary to determine his epistemological position. It is not sufficient, because it is strongly coupled with another characteristic. "Scientific realism" is linked to what Bachelard calls "discursive idealism" [21], another name for "open rationalism" [22]and which corresponds to a break with "immediate idealism"[23], that is to say with the consideration from an immutable reason to the benefit of a constantly creative mind of its own structures to adjust to reality; especially mathematical structures. Open rationalism also translates the quality of a scientific reason that flourishes only in its work of application to the diversity of its fields of study. It is rationalism that becomes "regional".

Moreover, this characteristic is not primary, because it results from a conquest. To follow this conquest is precisely to follow the emergence and promotion of this couple which, according to Bachelard, adequately describes the scientific spirit in its activity of knowledge and is fixed in the expressions of "applied rationalism" and "technical materialism or "technical realism"[24].

Bachelard's mathematical philosophy therefore proceeds logically from the characterization of science as the realization of the rational or the laborious application of the mathematical possible: "the application of scientific thought seems to us essentially realizing. We will therefore try to show […] what we will call the realization of the rational or more generally the realization of the mathematical" [25]. Bachelard's mathematical philosophy, which already takes shape in *Essay on Approximate Knowledge* , in addition to proceeding from the actual practice of mathematicians in arithmetic, algebra, analysis or geometry, is a stand against "ontological realism" in favor of a "constructed realism" and that he "metaphorical realism". It is "metaphorical" because "the reality of mathematics resides in their virtuality, that is to say their independence with regard to the concrete cases in which they are embodied" [26].

This "metaphorical realism" feeds on the pre-intuitionist theses of French mathematicians from the beginning of the 20th [century] , such as Borel and Baire; theses according to which only entities that can be calculated or approximated by a calculation are acceptable in mathematics. Bachelard's mathematical realism is constructed in that he considers mathematics as resulting from a work of rational construction, and which, beyond their simple heuristic function, also has an ontological significance. Bachelard is however opposed, let us say it again, to "ontological realism", "realism of mathematical essences" inherited from Plato and which one finds in famous mathematicians such as Cantor, Gödel, or even Hermite, to whom Bachelard devotes a few analyzes in *Essay on Approximate Knowledge* . In his article "Noumene and microphysics" also, Bachelard makes it known that:

> "By its psychologically dynamic and inventive value, mathematical realism, as it results from its relations with contemporary physics, goes far beyond the completely Platonic sense in which the realism of Hermit was still placed" [27].

The fact is that for the Platonist Hermit,

> "Mathematical being is, in a way, static […] On the contrary, the reality of mathematical physics is enriched by a double dynamism: by studying it, we have as much chance of discovering phenomena as theorems. Moreover, it is always necessary to come to realize the theorems thus discovered.[28]

Here, Bachelard opposes the stable mathematical realism of the Platonists with an active and dynamic mathematical realism which, refusing dualism or the separation between theorems and phenomena, reason and experience, mathematics and the physical sciences, instead makes mathematics l element that allows phenomena to be constituted. The relation of mathematics to other disciplines must then be a relation of constitution. Bachelard is critically part of a movement, that of a mathematical activity which is no longer considered and considered outside of its relationship to physics and chemistry. This need to no longer cut off mathematics from other sciences of reality led Bachelard to reject the *formalism* of pure ideas and *the empiricism* of the concrete object.

At this level, we realize that Bachelard's mathematical thought breaks the abstract/concrete, mathematics/physical sciences dualism and *opens up* mathematics to the sciences of matter. This is not without

---

[20]Gaston Bachelard, *The Philosophy of No* , *op. cit.*, p.86.
[21]See "Discursive idealism", in *Etudes* , p.87.
[22]See " Surrationalism ", in *Etudes* , p.12.
[23]*Ibid.*, p.91.
[24]Gaston Bachelard, *The New Scientific Spirit, op. cit.*, p. 9.
[25]*Ibid.*, p.8.
[26]Vincent Bontems, *Bachelard*, Paris, Belles Lettres, 2010, p. 59.
[27]Gaston Bachelard, "Noumene and microphysics" in *Etudes, op. cit.*, p.17.
[28]Same.

the detour of an allegiance to Kantianism: "The criticalist philosophy, whose solidity we will underline, must be modified according to this openness" [29]. Bachelardian mathematics achieves the synthesis of intellectualism and empiricism. Kant rejected pure intellectualism and radical empiricism, the categorical opposition of which leads to a split between rational truths and empirical work [30]. Kant, posing the necessary articulation of reason to experience, refuses to cut off mathematical work from its application to sensible reality. Rational science is no longer constituted in total independence with regard to experience. In this respect, Bachelard's expression of "applied rationalism" is very significant, an expression which clearly translates the fact that science for Bachelard is an applied mathematics. There is therefore no longer a break, in the chain of sciences, between mathematics and the sciences of reality.

This idea is clearly expressed by one of Bachelard's masters, Léon Brunschvicg, for whom a physics without mathematics is only a mediation on qualities, while a mathematics which does not apply to study of reality remains superficial. Commenting on this specific point in Brunschvicg's thought, Robert Blanché asserts:

> Just as, he said, a physics lacking the intellectual framework that only mathematics can provide would not rise above the level of a speculation on qualities, so too in another sense, an arithmetic which would not already be a physico-arithmetic discipline, that is to say whose propositions would not remain, directly or indirectly, in connection with reality, would not deserve the name of science.[31]

Ferdinand Gonseth, with whom Bachelard has many theoretical affinities, also refuses to admit that the work of mathematics is carried out at a distance from experience and independently of it. The latter refuses the break between mathematics and the sciences of reality. There is no pure abstract, nor autonomous concrete. The abstract is only justified in its act of constituting the concrete. In other words, the abstract can only be conceived when engaged in a certain realization of the concrete. As such,

> "Physics is abstract relative to sensory data, but concrete relative to mathematics, which itself will become concrete for logic; and logic itself, in its abstract generality, is ultimately only a "physics of any object." For the system of logical laws is not fixed *ne varietur* and once and for all, it is susceptible to modification and refinement by opening up to an experience that is never complete. »[32]

The evocation of these positions or theses helps us to better identify the ideas that contributed to mature the originality of Bachelard's position. In *Applied Rationalism* , he defends the necessary collaboration of reason and experience, the sciences of reality now only having to be constituted by the detour of a rationalization or mathematization of experience, the only guarantee of access to the apodicticity. However, he proposes, instead of architectonic rationalism based definitively on stable and immutable principles, an inductive and subversive mathematical rationalism because it is active and polemical: "Rationalism in action in the physical sciences cannot be confused with an elementary rationalism, immobilized in the universality of principles. Its role is not limited to summarizing experiences".[33]

It is a rationalism which, elevated above measure and flourishing in calculation, must constantly revisit its frameworks and principles to adequately interpret experience while also readjusting them so that they retain their operability.

By this double consideration of an active, creative and creative reason which draws its instruction from its dialogue with experience, and from an experience itself dynamic, supposed to control the work of reason, the gap which maintained the gap between mathematics and the sciences of reality. The dialogue is re-established between reason and experience, mathematics and reality, rational and empirical, abstract and concrete, form and content. This is the reason why the question of the interdisciplinarity of mathematics is directly consecutive to the application of rational work.

## IV. CONCLUSION

Our fundamental concern was to characterize the mathematical philosophy of Gaston Bachelard. It seemed to us quite legitimate to characterize it as mathematical realism insofar as it is a "realization of the rational in physical experience" [34], "or more generally the realization of the mathematical" [35]And since *the sense of epistemological vector goes from the rational to the* real [36], it is mathematics which makes it possible to infer,

[29]Gaston Bachelard, *The Philosophy of No,* op. quoted,p.11.
[30]See Immanuel Kant, *Critique of Pure Reason*, tr. by A. Tremesaygues and B. Pacaud, 11th ed., Paris, Quadrige/PUF, 1986.
[31]Robert Blanché, *Epistemology*, Paris, PUF/ "What do I know? », 1972, p. 84.
[32]*Ibid.*, p. 85.
[33]Gaston Bachelard, *The Rationalist Engagement*, *op. cit.*, p.94.
[34]Gaston Bachelard, *The New Scientific Spirit*, *op. cit.*, p. 9.
[35]*Ibid.*, p. 8.
[36]Same.

by induction, the existence of physical realities. We have thus been led to observe the particular and privileged status accorded by our author to mathematics. Mathematics is for Bachelard purveyors of rationality in science. Science derives its rationality and dynamism from its mathematical roots. Better still, it is mathematics that constitutes the condition of the possibility of science, in particular of the physical and chemical sciences. Bachelard thus invites us to consider abstraction as the appropriate and fruitful approach of the scientific spirit. Mathematical thought forms the basis for the explanation of physical phenomena. Mathematics has an inductive value in that it makes it possible to infer the existence of physical realities. It is mathematics that gives objects their reality, aided in this by technical instruments which themselves are “materialized theories”. Scientific experimentation is a work of technical realization of mathematical noumena. And when Bachelard speaks of mathematics, he manifests a strong attraction for algebra, which, beyond simple measurement, is a true source of thought, calculation, invention, inference. Science for Bachelard would therefore only be applied mathematics.

## REFERENCES


[1] D. Lecourt, *The Philosophy of Science*, Paris, PUF, “Que sais-je?”, 2001.

[2] E. Meyerson, *Identity and Reality*,Paris, Alcan, 1908.

[3] I. Kant, *Critique of Pure Reason*, tr. by A. Tremesaygues and B. Pacaud, 11th ed., Paris, Quadrige/PUF, 1986.

[4] G. Bachelard, *Essay on Approximate Knowledge* (1928), Paris, Vrin, 1973. *Applied Rationalism*,Paris, Quadrige/PUF, 1949. *The rationalist activity of contemporary physics*, Paris, PUF, 195), *The new scientific spirit* (1934), Paris, PUF,1984.

[5] *The philosophy of no* (1940),Paris, PUF, Quadrige, 1981.

[6] “Noumenon and Microphysics”,in *Studies*, Paris, Vrin, 1970.

[7] J. Chatué et Al., *General Introduction to Epistemology*, Dschang University Press, 2020.

[8] Plato, *The Republic*, *Paris*, trans. Robert Baccou,Garnier Flammarion, 1966.

[9] R. Blanché, *Epistemology*, Paris, PUF/ “What do I know? », 1972.

[10] Vincent Bontems, *Bachelard*, Paris, Belles Lettres, 2010.