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Multiply Connected Topological Sociology, and Some Applications of Mathematical Methods in Social Sciences

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ABSTRACT: First, we develop the multi-connected topological economics to the multi-connected topological sociology, etc. Next, we research the social field (including general relativity) and some applications. Third, the complex analysis is discussed. Fourth, variational calculus is studied. Fifth, we search nonlinearity and a new mathematical relation in social sciences. The multi-connected topology, general relativity and other mathematical ways will open new windows for investigations of economy and sociology, etc.
KEY WORDS: *social science, economics, sociology, topology, field, complex analysis, variational calculus, general relativity.*

I. INTRODUCTION

It is a developing trend that various social sciences are quantitatively described by different mathematical methods. In economics, this includes the famous input-output model, household consumption choices, preferences, maximizing utility, etc [1].

Economy and society are closely related, which is researched from M. Weber to T. Parsons and N. Smelser. So the economic sociology and its basic principles are proposed by Granovetter and Swedberg, et al [2-5]. Mantegna and Stanley searched econophysics [5]. Kirman, and Arthur, et al., studied the complex economics [6,7]. Certain statistical aspects of social systems are described by appropriately defined quantities named social potentials. Relations between social potentials are postulated by drawing an analogy with thermodynamics relations between thermodynamic potentials, thus obtaining a toy model of some of the statistical properties of social systems. Stepanic, et al., presented an approach to a quantitative description of social systems, and interpreted a socially relevant acting that does not invoke structural changes in social systems [8]. In this paper, we propose the multi-connected topological sociology, and discuss the social field, the complex analysis and variational calculus, etc., in social sciences.

II. MULTIPLY CONNECTED TOPOLOGICAL SOCIOLOGY

Using the similar formulas of the preference relation and the utility function, we proposed the confidence relations and the corresponding influence functions that represent various interacting strengths of different families, cliques and systems of organization. Since they can affect products, profit, prices, and so on in an economic system, and are usually independent of economic results, therefore, the system can produce a multiply connected topological economics [9-12]. The political economy is an economy chaperoned polity, it must produce consequentially a binary economy. When the changes of the product and the influence are independent one another, they may be a node or saddle point. When the influence function large enough achieves a certain threshold value, it will form a wormhole with loss of capital. Various powers produce usually the economic wormhole and various corruptions. This has the fractal structure. We propose the binary periods of the political economy by the complex function and the elliptic functions.

Further, it may develop to the multi-connected topological sociology [13,14], and the multi-connected topology politics, multi-connected topology ecology and environmental science, etc. Their main mathematics is the same basic formulas and the complex variable function, etc.

In a multiply connected region of topology there is a famous Euler-Poincare formula

$$\sum_{m=1}^{n}(-1)^m a_m = \sum_{m=1}^{n}(-1)^m p_m . \qquad (1)$$

For a convex polyhedron, a_0, a_1, a_2 (V,E,F) denote the number of vertices, edges, and faces, respectively; p_m is the mth Betti number of complex K. This may be considered intuitively as the numbers of m-dimensional holes in K, or is the number of (m+1)-dimensional chains that must be added to K so that every free m-cycle on K is a boundary [15]. The number $\sum_{m=1}^{n}(-1)^m a_m = k$ is called the Euler characteristic of the complex K. In the polyhedron,

$$V - E + F = 2(1-g)\,. \qquad (2)$$

Here g is the genus of a curved surface.

Assume that vertices represent the number of market, which is direct proportional to the sales volume *y* and the profit, and edges represent the market network. But the multiply connected economy brings the profit decrease. In this case there is a defective profit due to the genus *p*. Further, it may be related to the knot polynomials [16].

According to the topology, the world economy can be divided into two big categories: 1. Single connected topological economy, corresponding to the market economy. 2. Multi-connected topological economy, and corresponds to the political economy, etc.

This can explain a variety of multi-centers and multi-systems, which are related or irrelevant with economics.

Genus g involves the number and size of the holes. Any topological economy and society with porous and bigger holes are hard to sustain. Multi-connectivity topology forms multi-centers, which will seriously affect the effectiveness of a single center.

In the multi-connected topological economics and sociology, we may first introduce the total economic value S_t and the free (available) economic value

$$S_{eff} = S_t - (S_{lw} + S_{dw})\,. \qquad (3)$$

S_{eff} is S_t subtract the social waste, which equals two parts: 1) the necessary (bright) loss S_{lw} includes various transparent social welfare, government spending, education fee, defense budgets, etc; 2) dark corruption depletion S_{dw}. From this we introduce various ratios of various topologies, S_{eff}/S_t and so on. While S_{lw}/S_t is a transparency index of society, and S_{dw}/S_t is a dark index of society. It is related to the social Gini index.

Any dictatorship has only one purpose: it is all designed to maintain its individual rights. This would create a huge black hole, which could modify and abolish the basic rules of the economy and society, even launch the war. Today Russia and Ukraine are both big grain producers, but serious famine occurred from man-made disasters during the Soviet Union.

III. SOCIAL FIELD

Lewin discussed field theory in social science [17]. Wilkinson proposed the community as a social field [18]. Social fields may be the microscopic fields or the macroscopic fields. Both is related each other [19,20]. Helbing discussed the social field not only as an external environmental factor, and also as a mathematical model of individual interaction on individual behavior [21]. Bourdieu discussed the economic field [22]. Levitt, et al., discussed the transnational social field perspective on society [23]. Rawolle studied the cross-field effect and temporary social field of recent Australian knowledge economy policies [24].

The interactions between social members, social system and environment form different social fields on politics, economy, society, culture, religion and so on [25,26].

In various social fields, gradient of the scalar field φ is a vector field:

$$grad\varphi = \nabla\varphi = \sum_{i=1}^{n}\frac{\partial\varphi}{\partial x_i}\vec{i}\,. \qquad (4)$$

For example, human income is the scalar field, whose gradient may describe difference of total citizens. If gradient of stable circumstance and society is too big, stability of these systems will be destroyed. Gradient of the tensor field A of n order is a tensor field of n+1 order:

$$gradA = \nabla A = \{\frac{\partial A_{ij}}{\partial x_k}\}\,. \qquad (5)$$

Divergence of the vector field $\vec{A}$ is a scalar field. It may describe loss of capital, etc. Divergence of the tensor field A of n order is a tensor field of n-1 order:

$$div\vec{A} = \nabla \cdot \vec{A} = \{\frac{\partial A_{ij}}{\partial x_i}\}. \qquad (6)$$

Rotation of the vector field $\vec{A}$ is still a vector field. Further, they may be various social tensor fields, spinor fields, twistor fields and superposition of many fields.

The whole social field and ecological field form the special time-space. This is similar to the relativity. In general relativity the basic equations are:

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -kT_{\mu\nu}. \qquad (7)$$

Here $G_{\mu\nu}, T_{\mu\nu}$ are the space-time curvature and the energy-momentum tensor, respectively. Two aspects affect each other (Fig.1).

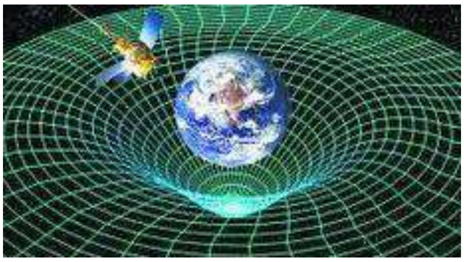

Fig.1. Matter and movement determine the space-time, and space-time determines the evolutional orbits.

We cannot fight against nature and its laws. "The era produces their heroes, and heroes produce their era." Era and heroes combine each other, which will form a magnificent history [27]. This exhibits unification between inevitability and chanciness in history. The era is big surroundings and conditions of historical evolution, while chance and hero, etc., are various occasional factors of happened historical events [28]. In Coleman-Granovetter collective-individual relations, the former is the collective state determines the individual state, and the latter is the individual outcome determines the collective outcome. General relativity is the same with the relation between cultural tradition as a big background and society-economy [4]. In Fig. 1 the big mass of the center and its movement correspond to the great countries and great men that determine space-time and era, from which everyone's mass and efforts determine the orbits of life. Both determine the evolution of whole society and mankind. This as a universal physical representation of causality is a great contribution of general relativity to modern social science [29,30]. It is the causality field as a common basis of various natural sciences, Buddhism and some social sciences.

From Eqs.(7) a wormhole (Fig.2) or a black hole may be formed [29,30], the original social system will be alienated and badly deformed. It also corresponds to the structural holes [31], which and Einstein-Rosen bridge extended namely correspond to social networks [32].

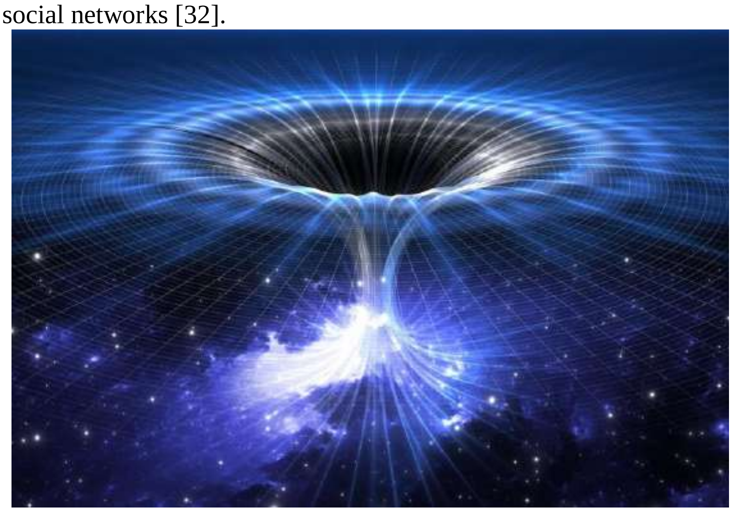

Fig.2. Wormhole.

IV. COMPLEX ANALYSIS IN ECONOMICS-SOCIOLOGY

Economics seems to be similar to the affine geometry x→Tx+b, which changes size and translation. The complex analysis corresponds to the plane, and has many features, such as there are two cycles. For a complex smooth function $f(z)=g+ih$, it satisfies the Cauchy-Riemann equations:

$$\frac{\partial g}{\partial x}=\frac{\partial h}{\partial y},\ \frac{\partial g}{\partial y}=-\frac{\partial h}{\partial x}. \quad (8)$$

A complex system may be described by the complex functions, which obtain often multivalue. For instance, a function of single value e^{nx} is a logarithmic spiral curve, and corresponds to an exponential economic growth. It develops to a complex function:

$$e^{inx}=\cos nx+i\sin nx. \quad (9)$$

This is a multivalue function, and corresponds to the periodic economic growth. A complex equation $p(\alpha,y)=0$ is n-th order of y, for a plane of $x=\alpha$, there are the n-layers curved surfaces of y.

If the political economy is an economy chaperoned polity, it will produce consequentially a binary economy. The political economy is usually imperfect economic question, even completely is not an economic question for some particular cases. It is not a strict economic rule, because in this case economy is only an appendage of polity. The economy will change along with polity.

The political economy as a multiply connected topological economics can be described by the complex function and the elliptic functions, which have two periods of economy and policy. The complex function corresponds to a complex social system of economy and polity. In the multiply connected topological economy, economy corresponds to a real part, and policy and relation, etc., correspond to an imaginary part. A complex plane corresponds to the surface of the Riemann sphere, which is called a stereographic projection [33].

An elliptic function is a meromorphic function on the complex plane [34], and has the double periods:

$$f(z+m\omega+n\omega')=f(z). \quad (10)$$

Here ω and ω' are the two different basic periods. In this case, the economical development will possess two periods of economy and policy, respectively. It includes the Weierstrass elliptic function:

$$W(z)=\frac{1}{z^2}+\sum_{\omega\in L}[\frac{1}{(z-\omega)^2}-\frac{1}{\omega^2}], \quad (11)$$

where the sum is taken over the set of all non-zero periods, denoted by L [34]. Its expansion shows different influence degrees of economy or policy, for example, the confidence relation for different families, cliques and systems of organization [9-11]. There is a theorem [34]: Assume that the elliptic function *f* has no poles on its boundary, then the sum of the residues of *f* is 0, i.e.

$$2\pi i\sum \mathrm{Re}\, sf=\int_{\partial P} f(z)dz=0. \quad (12)$$

It may describe a special case: Economy and policy cancel out, so economy cannot be developed. Its corollary is: An elliptic function has at least two poles on the torus, i.e., the two centers of economy and policy exist simultaneously. The elliptic curves with singular invariants have complex multiplication from an imaginary quadratic field.

The Weierstrass zeta function is [34]:

$$\varsigma(z,L)=\varsigma(z)=\frac{1}{z}+\sum_{\omega\in L}[\frac{1}{z-\omega}+\frac{1}{\omega}+\frac{z}{\omega^2}]. \quad (13)$$

There have $\varsigma'=-W(z)$ and

$$\varsigma(\lambda z,\lambda L)=\frac{1}{\lambda}\varsigma(z,L). \quad (14)$$

In this case it is a scaling invariance, and has fractal. There is a constant $\eta(\omega)$ such that

$$\varsigma(z+\omega)=\varsigma(z)+\eta(\omega). \quad (15)$$

Their separation is namely a connected graph becomes the non-connected graphs [35,36] in graph theory of the political economy.

Moreover, Granovetter complex thinking [4] corresponds to the complex function and also to general relativity.

Complex number forms a lattice in the plane. The complex analysis can correspond to the changing lattice. Function can be generalized to higher dimensions, and the calculus of quaternions, octonions [37], matrix and other functions. They will be more complex, such as $f(x,iy,jz,kw)$, so there have $\frac{\partial f}{\partial x},\frac{\partial f}{\partial y},\frac{\partial f}{\partial z},\frac{\partial f}{\partial w}$, etc.

V. VARIATIONAL CALCULUS

Variational calculus derived extreme values, and economics and various social sciences are often constraints some conditions, i.e.

$$H(x_i,x_i')=F(x_i,x_i')+\lambda G(x_i)\,. \quad (16)$$

For the reinvestment of commodities, the output rate of per unit time is q (t), and the growth rate $dq/dt=\alpha uq$ is proportional to the percentage of reinvestment u(t). The selected control variable u(t) reaches the maximum value of total commodity in the market within time [0, T], where q is the state variable. The target functional is:

$$J(u,q)=\int_0^T[1-u(t)]q(t)dt\,. \quad (17)$$

Hamiltonian is

$$H=-L+\lambda F=(1-u)q+\lambda(\alpha uq)\,. \quad (18)$$

The control variable u should switch at time $\alpha\lambda(t)-1=0$, i.e., $t_s=T-(1/\alpha)$. This is related to a new development direction [9,12].

$$J=q_0e^{\alpha t_s}(T-t_s)\,. \quad (19)$$

Ramsey's economic growth model (1928) proposed the social consumption utility W (c) maximization under the constraint:

$$\frac{dk}{dt}=f(k)-\alpha k-c,k(0)=k_0\,. \quad (20)$$

Here c is the consumption per person, and k is the total capital per person.

$$W(c)=\int_0^\infty e^{-pt}u(c(t))dt\,. \quad (21)$$

Hamiltonian is

$$H(k,\lambda,c)=u(c)e^{-pt}+\lambda(f(k)-\alpha k-c)\,. \quad (22)$$

The optimal condition is:

$$\frac{\partial H}{\partial c}=u'(c)e^{-pt}-\lambda=0\,. \quad (23)$$

In 1965 it is developed to Ramsey-Cass-Koopmans model [38].

Assume that general energy is E=T+V, and Lagrangian is $L=T-V$. Potential has usually cycle, from the economy and capital (i.e., the bottleneck of development) to power, life and energy. Further, the form and connotation of V must be developed. Let

$$L=\frac{1}{2}\dot{u}^2-V(t,u)\,. \quad (24)$$

Equation $\ddot{u}+V_u(t,u)=0$ is the nonlinear vibration equation. The general periodic solutions are derived from the variational method [39-41]. They are homoclinic or heteroclinic orbits. For potential

$$V(t,u)=-\frac{1}{2}|u|^2+a(t)|u|^\mu\,. \quad (25)$$

When $\mu>2$, equation has from non-trivial solution to heteroclinic orbit.

Variational method applies to economic and social sciences, and obtains Euler-Lagrange equation and Lagrange equation, and Hamilton equation. Then it may change to the general mechanical form of economics and social science. For different topological systems there are different Lagrangians and Euler-Lagrange equations, which have different maximums and periods.

We introduce kinetic energy, potential energy and Noether theorem and its extension. If space and time are uniform and stable, energy, momentum, matter flow and capital flow will be conservation. On the contrary, it is not conserved. The continuity equation is generally established:

$$\frac{\partial\rho}{\partial x_\mu}=cj_\mu\,. \quad (26)$$

Here j_{μ} is an injected material-capital flow, etc. This is similar to fluid dynamics, which is more complex, and can form obstruction, nonlinear turbulence, chaos, etc., and correspond to the economic or social crisis.

Band energy corresponds to the intellectual class. Religion and cults are both potentials and forces. Inequality of social wealth and income distribution implies the social crises, and are related to revolutions. Once the accumulation value of conversion variables reaches a certain degree, the revolution is very likely to occur [42-44].

VI. APPLICATIONS OF NONLINEARITY AND NEW MATHEMATICAL RELATION

Humanity as an inseparable whole on Earth possesses common environment and benefit. In social sciences various nonlinearities exist widely, which include chaos, fractal and soliton. Based on the inseparability and correlativity of the social systems, we researched the nonlinear whole sociology and the four basic laws [25]. We proposed new nonlinear theory of economic growth and its three laws: Economic takeoff-growth-stagnancy law, social conservation and economic decay law, and economic growth mode transition and new developed period law. A corresponding figure is represented [25]. The social open-reform is a necessary and sufficient condition for further economic development. Based on the main characteristics of knowledge economy, the four theorems on the knowledge economic theory are proposed, and the production function and basic equations are expounded [45]. Some possible directions of the development on the knowledge economy and a sustainable development theory of new economics are discussed.

In 1983 Weidlich and Haag published book *Concepts and Models of a Quantitative Sociology. The Dynamics of Interacting Populations* [46]. Then Lefebvre proposed the production of space [47]. Zohar and Marshall applied quantum physics, and proposed the quantum society [48]. Mol and Law [49] discussed regions, networks and fluids, in which there is wave-particle duality, and may apply to city, and develop to global wave and particle, and globally networks and fluids [50]. Various networks correspond to the social topology [13].

For two any sets M and N, we researched the simple mathematical joint relation of algebra in set theory[51]:

$$(M,k)(k',N)=(M,kk',N). \qquad (27)$$

Here k and k' may be the same or different operators or quantities, for example, number, scale, vector, tensor, matrix, etc. And $kk'=j$ is a joint relation between M and N, which is but only those known union, intersection, difference, and so on. This may be addition, product, etc., and may indeed be part addition and product, i.e., an incomplete joint. It is a type of conglutination or adhesion joints. Complete joint for k and k' with the same types is called the fastener joint. It may describe various relations among sciences and many intersecting sciences [51]. This is applied to various aspects in natural science and social science, which includes transcription, splicing and replication, etc., in biology, and both main relations in micro-economics and macro-economics, etc. Further, it may be corrected and developed.

This can be applied to accurately determined economics, such as some specific points of action in economics, in which concentrated investment in capital, manpower, technology, etc. For the macroeconomics it may be combined with the input-output model.

These need points can derive maximize happiness. It includes the hierarchical theory, different needs for different levels and different classes.

It is impossible to have only one equilibrium state in economics, because the equilibrium state depends on the interaction and the structure. Brian Arthur demonstrated that economics is a path-dependent theory. In 1990s P. Malaney and E. Weinstein proposed that a mathematical representation of the path-dependence system is the gauge fields. Further, it is the integration of economics and ecology, and both developments, which include complex open systems, continuous evolution over time, path dependence, existence of multiple equilibrium states, and constrained by feedback mechanisms [52]. The key is that the time and the evolution must be considered.

In micro-economics a main relation is expressed by $M(\genfrac{}{}{0pt}{}{q}{p},E_{mi})$, here M is market, q is quality and p is price. In macro-economics a main relation is expressed by $I(e,E_{ma})$, here I is income and e is employment. Governmental action is by G(p,E) here G is government and p is policy.

The multiply connected topological economics may be expressed by c(p,M)=(M,k)d, here c is confidence relation, p is power and d is a dark system separated. Society may be various states, but generally can not be gaseous state. And the solid society possesses bigger defects, and which diffuse easily, in particular, a pyramid structure is unsustainable. Modern society and persons are more analogy with liquid, therefore, we applied the social hydrodynamics [11]. Using the nonlinear equations of hydrodynamics we researched the formulations of the binary and multiple centers in various social systems [53]. It is expressed by:

$$(M,k)(k',N) = \begin{pmatrix} C_1 \\ C_2 \end{pmatrix} \to \begin{bmatrix} C_1 \\ C_2 \\ \vdots \\ C_n \end{bmatrix}. \tag{28}$$

In a word, the multi-connected topology, general relativity and other mathematical ways open new windows for investigations of economy and sociology.

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