

Extensive Quantum Economics: Its Theory, Equations and Applications

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ABSTRACT: First, the extensive quantum theory is researched. Second, Orrell's quantum economics is introduced. Third, we propose the extensive quantum economics and its three fundamental principles: duality, uncertainty principle and extensive statistics. Fourth, we may apply quantum principles, theory and mathematical equations to economics. The corresponding Schrödinger equation and its potentials are studied, in which different potentials correspond to different economic policies and development models in quantum economics, and form different results and varying energy levels. Fifth, we discuss some economic models, such as the nonlinear theory of economic growth and the multiply connected topological economics, etc. Finally, we propose a specific prediction method. Since quantum theory has rich mathematical and physical contents, so the extensive quantum economics not only may be applied to many economic regions, and theory may be corrected and developed.

KEY WORDS: economics, quantum, principle, theory, equation, model.

I. INTRODUCTION

Great physicist Feynman pointed out: "There are certain situations in which the peculiarities of quantum mechanics can come out in a special way on large scale." In a special situation "quantum mechanics will produce its own characteristic effects on a large or 'macroscopic' scale" [1]. We proposed the extensive quantum theory [2] and its three laws: 1. Extensive quantum is its element in any system. 2. Its theory has similar quantum formulations with different quantum constants H . 3. Evolutions of systems may be continuous, but stable states are quantized. Its mathematical base is fractal [3]. This is not only microscopic, but also applicable to the macro fields. In 2025 Nobel Prize in Physics three winner J. Clarke, M.H. Devoret and J.M. Martinis affirmed the existence of macroscopic quantum effects.

We assume that the extensive Schrödinger equation is:

$$iH \frac{\partial \psi}{\partial t} = -\frac{1}{2m} H^2 \nabla^2 \psi + V\psi. \quad (1)$$

Its time-independent Schrödinger equation ($H=1$) is:

$$\frac{d^2 \psi}{dr^2} + 2m(E - V)\psi = 0. \quad (2)$$

In this paper based on the extensive quantum theory and the Orrell's quantum economics, we propose the extensive quantum economics and its fundamental principles, and discuss Schrödinger equation and some applications.

II. ORRELL'S QUANTUM ECONOMICS

Orrell argued that economics sees itself as the science of scarcity, but instead it should be the science of money. Orrell maintains that the economy itself is a quantum system, and proposed the quantum economics, whose research shows that money exhibits quantum properties such as duality and entanglement, etc. Building on this, quantum economics emerges as a new science, offering novel interpretations of fundamental concepts including value, debt, risk, decision-making, environmental factors, and even happiness, ethics and so on. He transmutes money into a quantum phenomenon, and the economy is an archetypal example of a quantum social system [4].

Economics should begin by analysing the properties of money-based transactions, which like quantum entities have a fundamentally dualistic nature. By building on ideas from quantum money, quantum finance and quantum social science, it shows that the economy is an archetypal example of a quantum social system,

complete with its own versions of measurement uncertainty, entanglement, and so on [5]. Further, Orrell researched quantum economics and finance, and applied mathematics [6].

In Quantum Bayesianism (QBism) the wave function represents the degree of a subjective belief, and its collapse means that the subject updates his belief according to the information, which is completely consistent with the economic background [7].

III. EXTENSIVE QUANTUM ECONOMICS

The entire economic system exhibits profound quantum characteristics, governed by three fundamental principles: 1. Duality, individuals act as particles, while collectively forming waves [8]. Economic development and prices in market economy have all wave property. 2. The uncertainty principle [9,10] governs both macro and micro scales, with equilibrium being transient. Money is a classic measure of uncertainty [4]. 3. The extensive statistics. Many economic systems have randomness, for which the most similar physical theory is general quantum theory. Moreover, quantum entanglement may occasionally manifest.

Based on the analysis of the logical structure of quantum mechanics, we proposed that the wave-particle duality is the only basic principle of quantum mechanics. Statistics is the corresponding mathematical character. The other principles are all its physical or mathematical results, which includes the uncertainty principle [11].

In fact, the first application of quantum in sociology is the Bohr principle of complementarity. Zohar described bosons as “relationship particles” and fermions as “anti-social” [12]. Zohar and Marshall researched the quantum society from mind, physics as a new social vision [13]. Wendt studied quantum mind and social science, and proposed that people are the walking wave functions based on the causal closure (or completeness) of physics (CCP). This research core is that quantum theory can explain the phenomena of consciousness and intentionality, and an important basis is the quantum entanglement and macroscopic entanglement [14].

But, the some applications of quantum, such as sociology and economics, etc., have the scale question. We proposed that the extensive quantum theory [2,3] may break this limit, and apply to many aspects [15-18]. In quantum economics and quantum sociology the basis is that people and so on are an independent “quantum”, which corresponds to Leibnizian monads.

We researched and developed the social individual-wave duality. The social changes show a tendency after a tendency with the ideology-culture change and the science-technology developments, even if with some patterns. It may be related to the duality of structure [19] and the social quantum theory [20].

In history these tendencies include the Greek civilization, Confucius, Islam, the Renaissance, the Industrial Revolution, the Enlightenment, the French Revolution, Marxism, Leninism, Nazism, the Drastic Change of the East European Countries, many waves on freedom-democracy, network, information society, Arabia Spring, Fundamentalism and so on.

Postmodernism believes that social change is not always advancement [21], and is disorder, less-rule and chaos, even if is origin of a happenchance. It is a runaway world [19], and is a risk society [22], which includes many opposite dualities. Discourses power is a control force, but internet is an opposite energy. Author believes that human history is progress toward democracy, civilization and multi-cultures.

Individual behavior belongs to microeconomics, while overall distribution belongs to macroeconomics.

IV. APPLICATIONS OF QUANTUM EQUATIONS

We may apply quantum principles, theory and mathematical equations to economics. Different economic policies and development models correspond to different potentials in quantum economics, and form different results and varying energy levels. These potentials may correspond to different wells, centers, and evolve into multi-center systems analogous to multi-atomic molecules, i.e., the multiply connected topological economics.

Assume that economics is similar to the harmonic oscillator based on rotation invariance. Its potential is [23]:

$$V(r) = \frac{1}{2} Kr^2. \quad (3)$$

The quantum energy levels are:

$$E_n = H\nu\left(n + \frac{1}{2}\right), n=0,1,2,\dots \quad (4)$$

They can derive the quantization of various economic elements from product prices to businesses.

The nonlinear Schrödinger equation is:

$$iH \frac{\partial \psi}{\partial t} + \frac{1}{2} H^2 \Delta \psi + \frac{2m}{H^2} E \psi + k |\psi|^2 \psi = 0. \quad (5)$$

It is already applied to many aspects. Its soliton solution is [24]:

$$\psi = \psi_0 \operatorname{sech}[\sqrt{k/2} \psi_0 (x - u_e t)] \exp[i(u_e/2)(x - u_e t)]. \quad (6)$$

The corresponding time-independent Schrödinger equation ($H=1$) is:

$$\frac{d^2 \psi}{dr^2} + 4mE \psi + 2k \psi^3 = 0. \quad (7)$$

We develop Eq.(7) to new equation:

$$\frac{d^2 \psi}{dr^2} - a^2 \psi + 3a \psi^2 - 2\psi^3 = 0. \quad (8)$$

Integration once may become the logistic equation:

$$\frac{d\psi}{dr} = a\psi - \psi^2. \quad (9)$$

Its solution is:

$$\psi = \frac{a}{1 + c \exp(-ar)}. \quad (10)$$

It is a soliton of Γ -form. This corresponds to increasing limit in economics of natural resources, etc.

The Schrödinger equation with the linear potential $V=Fr$ is:

$$\frac{d^2 \psi}{dr^2} + 2m(E - Fr)\psi = 0. \quad (11)$$

Such Eq.(11) may become a Bessel equation:

$$\frac{d^2 \psi}{dz^2} + \frac{1}{z} \frac{d\psi}{dz} + \left(1 - \frac{\nu^2}{z^2}\right) \psi = 0. \quad (12)$$

Its solution is the Bessel functions (Fig.1), which shows also the wave form.

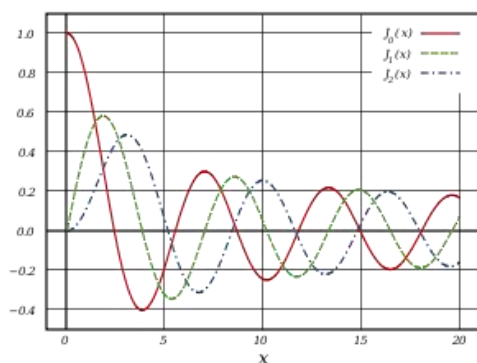


Fig.1 The Bessel functions

It is similar to the Lotka-Volterra model.

V. SOME ECONOMIC MODELS

In order to overcome a series of crises of the economic linear growth, based on Eq.(10) and the mathematical economics [25], we think that the nonlinear evolution is a universal rule for economic growth, and proposed the nonlinear theory of economic growth and its three laws [26]: First law: Economic takeoff-growth-stagnancy law. Any output and corresponding economic development all must pass a general nonlinear evolutionary process from takeoff to growth and stagnancy, no matter what for various merchandises or any country. This is unreasonable and impossible that anybody requests a persistent linear growth of economy. It corresponds to the maximum $\psi = a$ in Eq.(10) of the social effective throughput. This is a stagnancy dates of economic growth. Second law: Social conservation and economic decay law. For any society, since the original throughput outmoded gradually, the social ageing, the saturated marketplace; contrarily, employment, laborage, welfare, operating costs and so on will increase continuously, and add the resources consumed, the wastes raised, the environmental largeness press and so on. Such the corresponding social effective throughput and the original economy develop to a certain extremum, and will descend inevitably. Third law: Economic growth mode transition and new developed period law. Further development of social economy must exploits new merchandise and market, and adjust output configuration, and reform technique, and train personnel, so that boost up the immanence ability of social development and the international competitiveness. At the same time, the social framework and various personnel must readjust combination, and the management level raises up to follow the social development and new talented persons, new equipments, new outputs, new techniques and new capital introduced. Such the society should reform continuously to achieve a higher seedtime. This searches new economic growth point for microeconomics. It corresponds to a development of the paradigms in science.

The three laws on economic growth may be represented by Figure 2, in which CA expresses the first law, AA' expresses the second law, and AB expresses the third law. It should be a medium-time mode of economic growth, and is also three developed phases of social economy. Point A and dotted line are related with the limits to growth [27]. The third law connects to "the quality ladder" [28], which expresses a new period of development. The second law expresses a seasonal recession. They are two different foregrounds of economic evolution, and are two-bifurcation phenomena of nonlinear system. An infinite clone of the same developed mode will derive a disorder competition, and finally reach necessarily to chaos and economic crisis. Therefore, the nonlinear chaos economics is possibly related to the crisis economics.

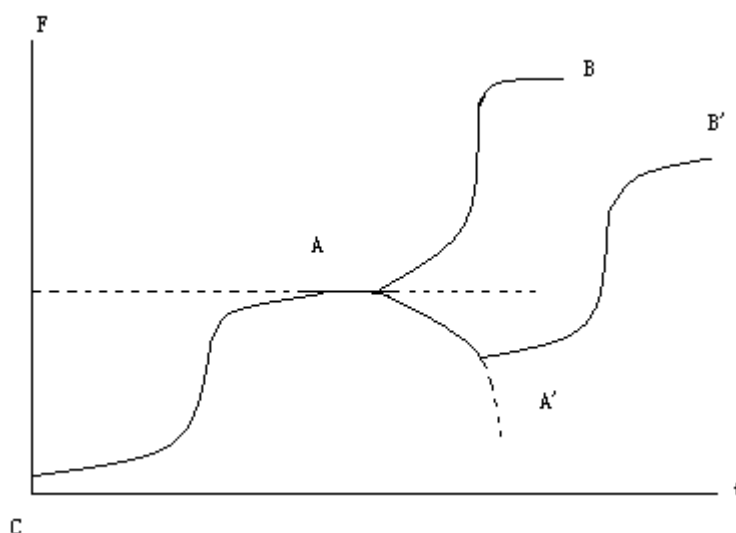


Fig.2 The three laws of economic evolution

Therefore, the social open is a necessary condition for economic further development, but it must add corresponding social reform as a sufficient condition of economic development. It may combine the theorem of transformation from energy to quality on social development [29,30]. Rivera-Batiz discussed the relation among democracy, governance and economic growth. Quantum leaps correspond to several big periods of economic growth in human history, such as the steam engine, the electric motor, the computer, and future AI.

The multi-center and multi-body systems are extremely complex, and the three-body problem has no solution. This is inevitably related to nonlinear relations, which can lead to chaos and chaotic economics. If the multi-body system is simplified to a two-body system, it will similar to the diatomic equation [23]:

$$\left(-\frac{1}{2M_1}H^2\nabla^2 - \frac{1}{2M_2}H^2\nabla^2 + VR\right)\psi = E_t\psi. \quad (13)$$

Its energy levels are:

$$E_{nL} = V + \left(n + \frac{1}{2}\right)Hv + \frac{L(L+1)}{2J}. \quad (14)$$

This determines the two quantum numbers of vibration n and rotation L .

The foundation of quantum economics is quantum sociology. The economic system is a particular social system, and should be whole and nonlinear. Using the similar formulas of the preference relation and the utility function, we proposed the confidence relations and the corresponding influence functions that represent various interacting strengths of different families, cliques and systems of organization. Since they can affect products, profit, prices, and so on in an economic system, and are usually independent of economic results, therefore, the system can produce a multiply connected topological economics [31-34]. If the political economy is an economy chaperoned polity, it will produce consequentially a binary economy. When the changes of the product and the influence are independent one another, they may be a node or saddle point. When the influence function large enough achieves a certain threshold value, it will form a wormhole with loss of capital. Various powers produce usually the economic wormhole and various corruptions. This is a mathematical application to economics, and has the fractal structure.

If the political economy is an economy chaperoned polity, it will produce consequentially a binary economy. The political economy is usually imperfect economic question, even completely is not an economic question for some particular cases. It is not a strict economic rule, because in this case economy is only an appendage of polity. The economy will change along with polity.

The political economy as a multiply connected topological economics can be described by the complex function and the elliptic functions, which have two periods of economy and policy. The complex function corresponds to a complex system of economy and polity. In the multiply connected topological economy, economy corresponds to a real part, and policy and relation, etc., correspond to an imaginary part. A complex plane corresponds to the surface of the Riemann sphere, which is called a stereographic projection [35].

An elliptic function is a meromorphic function on the complex plane [35], and has the double periods:

$$f(z + m\omega + n\omega') = f(z). \quad (15)$$

Here ω and ω' are the two different basic periods. In this case, the economical development will possess two periods of economy and policy, respectively. It includes the Weierstrass elliptic function:

$$W(z) = \frac{1}{z^2} + \sum_{\omega \in L} \left[\frac{1}{(z - \omega)^2} - \frac{1}{\omega^2} \right]. \quad (16)$$

Two systems and periods may be entangled.

A central force field may be Coulomb potential [23]:

$$V = -\frac{eQ}{r}. \quad (17)$$

Its energy levels are:

$$E_n = -\frac{m(eQ)^2}{2H^2n^2}. \quad (18)$$

They may be proportional to n , n^2 , or n^{-2} , for which $n \rightarrow \infty$, $E \rightarrow 0$ represent marginal productivity and development bottlenecks, etc.

For Eq.(1) we introduce the periodicity $V = A \sin \varphi$, which corresponds to the sine-Gordon equation. It may describe various economic periods and the cycled economics.

Moreover, Bessel functions (Fig.1) may also describe the cycled economy. For this our outlets are enterprising new energy sources, extending new living space and useful regions for humanity, expending new technology, and form a great industrial circulation like an agricultural ecological village. A Chinese traditional poem described vividly the circulation:

Fallen flowers are not merciless and useless

They transform manures, and enrich flowers.

(落红不是无情物，化作春泥更护花)

VI. DISCUSSION AND SUMMARY

Based on the main characteristics of knowledge economy, the four theorems on the knowledge economic theory are proposed, and the production function and basic equations are expounded. Some possible directions of the development on the knowledge economy and a sustainable development theory of new economics are discussed [32].

Statistics is usually used in econometrics. Different quantum statistics are used for different social systems, social compositions and interactions. We may research quantum statistics in social economics. Kings and certain positions are unique and necessarily incompatible, which need to use Fermi-Dirac statistics. In quantum economics the monopoly market agrees with Fermi-Dirac statistics, and the free market agrees with Bose-Einstein statistics.

In quantum economics we may apply the Feynman diagrams of quantum field theory, and its extension [36]. The invisible hand is namely quantum fluctuation. Probably, inflation corresponds to wave packet collapse.

Exact economic predictions are fundamentally impossible due to quantum fluctuations, the uncertainty principle, sensitivity to initial values caused by nonlinearity, and chaotic dynamics, etc. However, quantum mechanics has many solutions, so certain predictions can still be made for multiply connected topological economics, nonlinear economic growth theory, cycled economy, the knowledge economics [32] and so on.

For the extensive quantum economics we propose a specific prediction method: Assume that the Schrödinger equation is appropriate, and select the most correct potential V , then obtain the solution of this equation, whose square is the probability distribution, and may test the corresponding energy level.

In a word, based on the extensive quantum theory and Orrell's quantum economics [4-6], we propose the extensive quantum economics. Since quantum theory has rich mathematical and physical contents, so it not only may be applied to many economic regions, in which quantum information is directly related to the economy and even forms an important direction of new economic development, and theory may also be corrected and developed, for example, the superstring may be used to the quantum economics, in which high dimension will be related to the multiply connected topological economics [31-34].

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