

# Nonlinear Whole Economics and Its Applications

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**ABSTRACT:** First, we propose the nonlinear whole economics and its four basic hypotheses. Second, we research the extensive quantum economics. Third, based on the double solutions with soliton-chaos of nonlinear equations, we discuss the economic meanings: Soliton corresponds to the invariance in economic circulation, and chaos corresponds to corruption and economic crises, etc. The double solutions show the simultaneous existence on determinism and probabilism quantitatively in some nonlinear economic systems. Fourth, entropy economics and circular economics are studied. Fifth, we discuss development cycle and economic crisis. Based on the nonlinear economics, a conclusion is that the cycles are variable, such as political economics exhibits a dual-cycle pattern. Sixth, we search some social sustainable developed patterns. We must apply different methods to study complex economics.

**Key Words:** *economics, nonlinearity, quantum, soliton, chaos, entropy, cycle, crisis.*

## I. INTRODUCTION

In 1993 Drucker discussed that knowledge becomes a distinct resource, fundamentally transforming social structures, generating new socio-economic dynamics, and giving rise to new political frameworks, so ushering in a new information revolution in Post-Capitalist Society [1]. Floridi researched information society and its philosophy [2].

Paul Mason examines capitalism as a whole to identify its fundamental characteristics: capitalism is an organism with a life cycle; it is a complex system and a learning organism capable of continuously adapting to new changes, with survival instinct driving technological transformation. Mason argues that capitalism has reached the limit of its adaptive capacity, and that information technology, information goods, cooperative production, and networks will give rise to post-capitalism, so we can build a more equal society [3]. This represents an era of the information economy and a potential solution to crisis economics. In this paper, we propose the nonlinear whole economics, and discuss the economic meanings of soliton and chaos, and study entropy economics, circular economics, development cycle and economic crisis, etc.

## II. NONLINEAR WHOLE ECONOMICS AND ITS BASIC HYPOTHESES

Integrity and complexity are two fundamental characteristics of the economy. The unpredictability of economic crises vividly demonstrates the complexity of economic systems. A primary mathematical framework for describing this complexity is nonlinearity, which gives rise to chaos, and hallmark of crisis economics that often stems from nonlinear dynamics. Furthermore, nonlinearity gives rise to fractals, endowing economic systems of varying scales with self-similarity.

Based on the most basic features whole and nonlinearity of economics and combining the general nonlinear

theory, we propose the nonlinear whole economics, and four basic hypotheses:

First hypothesis: The inseparability and integrity exist always among different parts and different levels in various economic systems, which determinate the economic structures.

Second hypothesis: Many main characteristics of economic systems are produced from some especial structures of complex subsystems. From this the interaction and nonlinearity exist necessarily.

Third hypothesis: From individual and small households workshops to large enterprises and multinational corporations, various economic systems of different levels possess the totality and nonlinearity.

Fourth hypothesis: A basic property of any economic systems as an open system is this system and its environment must be a whole. Usually environment is regarded as a boundary condition of the system, but it and the economic systems have often various nonlinear relations.

This is similar to the nonlinear whole sociology [4] and the nonlinear whole ecology [5], etc.

The totality and the nonlinearity are two basic economic characters. They are closely related. Because of complexity, the inseparability, and the correlativity of the economic systems, their description must apply the nonlinear theory with the interaction terms.

Nonlinear economics includes nonlinear econometric models, which are especially useful for analyzing phenomena such as threshold effects, regime changes, or volatility clustering. By incorporating nonlinearity, researchers gain deeper insights into complex dynamics in macroeconomic policies, financial markets, and economic growth.

The basic mathematical model for the knowledge economy is nonlinear theory, in which the Cobb-Douglas production function [6,7] is:

$$Y = AK^{\alpha}L^{\beta} . \quad (1)$$

Synergetics has developed a nonlinear investor model that closely agrees with real-world [8]:

$$\frac{dX}{dt} = I + RX - CX^3 . \quad (2)$$

### III. EXTENSIVE QUANTUM ECONOMICS

Orrell proposed the quantum economics, whose research shows that money exhibits quantum properties such as duality and entanglement, etc [9]. Further, we combined the extensive quantum theory, and proposed the extensive quantum economics and its three fundamental principles: duality, uncertainty principle and extensive statistics [10]. Such we may apply quantum principles, theory and mathematical equations to economics. The corresponding Schrödinger equation and its potentials are studies, in which different potentials correspond to different economic policies and development models in quantum economics, and form different results and varying energy levels. We discussed some economic models, such as the nonlinear theory of economic growth, the multiply connected topological economics and so on, and proposed a specific prediction method. Further, we proposed wave-cycle and wave function, etc. According to the extensive quantum statistics, the extensive quantum economics should be divided three types: BE economics, FD economics, mixed economics. The input-output model corresponds to Heisenberg equation and S theory. We searched knowledge economics and its four theorems, and e-market, and discussed some other mathematical methods in economics [11]. Since quantum theory has rich mathematical and physical contents, so the extensive quantum economics not only may be applied to many economic regions, and theory may be corrected and developed. Human society and economics are very complex, and the extensive quantum economics only provides a new research method and a possible development direction.

The nonlinear Schrödinger equation is:

$$iH \frac{\partial \psi}{\partial t} + \frac{1}{2} H^2 \Delta \psi + \frac{2m}{H^2} E \psi + k |\psi|^2 \psi = 0. \quad (3)$$

It has the soliton solution. Its corresponding time-independent Schrödinger equation ( $H=1$ ) is:

$$\frac{d^2 \psi}{dr^2} + 4mE \psi + 2k \psi^3 = 0. \quad (4)$$

We develop Eq.(4) to new equation:

$$\frac{d^2 \psi}{dr^2} - a^2 \psi + 3a \psi^2 - 2\psi^3 = 0. \quad (5)$$

Integration once may become the logistic equation:

$$\frac{d\psi}{dr} = a \psi - \psi^2. \quad (6)$$

Its solution is:

$$\psi = \frac{a}{1 + \exp(ar)}. \quad (7)$$

Economic quantum entanglement must be described using nonlinear theory, and corresponds to globalization. It can be associated with the Kondratieff's long periods [3].

#### IV. DOUBLE SOLUTIONS OF NONLINEAR EQUATIONS AND ECONOMIC MEANING

The fundamental characteristics of soliton and chaos in nonlinear equation are completely different. But all nonlinear equations with a soliton solution may derive chaos. While only some equations with a chaos solution have a soliton. Soliton as new concept in applied science describes a pulse-like nonlinear wave with invariant, which is applied already in many ranges [12].

The nonlinear Schrödinger equation (3) may become:

$$\phi_{xx} + i\phi_t + k|\phi|^2 \phi = 0. \quad (8)$$

It has a soliton solution [12].

It may become the equation:

$$x' = \sqrt{a} \sin x. \quad (9)$$

Such this has the chaos solution [13]. For a stable state whose energy is  $H$ , if  $k=-b<0$ , the equation (3) will be:

$$\phi'' + H\phi - b\phi^3 = 0. \quad (10)$$

It may become a difference equation [13]:

$$X_{n+1} = 1 - \frac{H}{2} X_n^2. \quad (11)$$

It is a known equation, which has the chaos solution, and its parameter determined the bifurcation-chaos is  $\mu = H/2$ .

The nonlinear Dirac equation is:

$$\gamma_{\mu} \partial_{\mu} \psi + m \psi - l_0^2 \psi (\psi^+ \psi) = 0. \quad (12)$$

Let  $\eta = \alpha(\gamma_{\alpha} x_{\alpha} - u \gamma_0 t)$ , the equation is:

$$\frac{d\rho}{d\eta} = l_0^2 \rho (2\rho - 1), \quad (13)$$

whose solution is:

$$\rho = \frac{1}{2 - \exp(-l_0^2 \eta + c)}. \quad (14)$$

It is analogous to a soliton since  $\rho = (2 - e^c)^{-1} (\eta = 0)$  and  $\rho = 1/2 (\eta \rightarrow \infty)$ . Using a substitution

$\rho = (2 - l_0^2 x) / 8$  for Eq. (13), then the corresponding difference equation is

$$X_{n+1} = 1 - \frac{1}{4} l_0^4 X_n^2, \quad (15)$$

which has the chaos solution and the parameter  $\lambda = l_0^4 / 4$ .

According to the bifurcation-chaos theory and Eq.(15), assume that the replacement may represent corruptions from one level to another level, or continuous expansive corruption, this equation may be called to a corruption equation. For this, if corruption cannot be controlled effectively, the bifurcation with two periods will appear, and finally, the solution will achieve chaos, namely, whole social system will decompose and become confusion. From quantitative value of bifurcation-chaos, we may estimate that if the corruption members in any system or a certain level attain to 37.5%, a dangerous double periodic bifurcation will appear. If the members attain to 0.70057759...=70%, this system as whole will show an incurable dark. It is represented visually in a known figure from bifurcation to chaos (Fig.1).

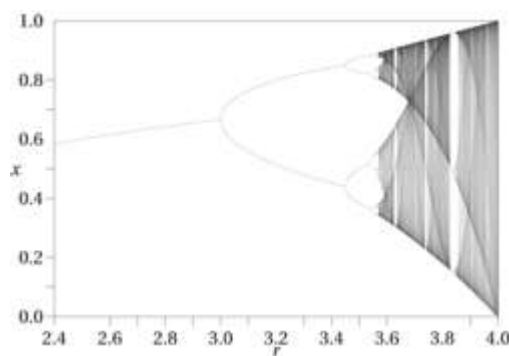


Fig.1 Bifurcation-chaos diagram

The cubic Klein-Gordon equation is:

$$\partial_{\mu}^2 \varphi - m^2 \varphi + a \varphi^3 = 0. \quad (16)$$

It has the simplest soliton with a kink shape, and the chaos solution [13].

The conditions of the two solutions are different. When some parameters are certain constants, the soliton

is derived; while these parameters vary in a certain region, the bifurcation-chaos appears. Therefore, the former corresponds to a stable state, and the latter is a changeable process [13].

Now we discuss some possible economic meanings of the nonlinear equations with the double solutions. Soliton corresponds to the invariance in economic circulation, such as currency in equivalent exchange, namely the Gold Standard under the Bretton Woods system, corresponds to stability. Chaos corresponds to fragility and sensitivity to initial conditions. To some extent, economic crises may correspond to chaos. The approach to overcoming economic crises lies in eliminating the conditions that give rise to chaos. This connection relates to chaos control, whose general methods may be applied to economic crises. By repeatedly applying carefully selected appropriate small perturbations to a system, one can maintain originally unstable trajectories as stabilized periodic motions. General chaos control can include four aspects [14-18]: harnessing chaos, generating chaos, narrow-sense chaos control, and chaotic synchronization.

The double solutions show the simultaneous existence on determinism and probabilism quantitatively from an aspect in some nonlinear economic systems.

## V. ENTROPY ECONOMICS AND CIRCULAR ECONOMICS

We understand the entropy law to be the most basic law of nature. It underpins physics, biology, mechanical systems. Such entropy economics is also base of society. Economic processes involve valuation and production decisions, requiring fixed investments, and that are inherently dynamic and unstable: there is no long-term equilibrium.

Georgescu-Roegen studied that the entropy law in the economic process imposes an absolute resource scarcity which cannot be overcome with technological change, exploration, or substitution [19]. Daly discussed the controlled throughput by physical method at the point of lower entropy in the economics of the steady state [20]. Swaney searched economics, ecology and entropy [21]. Zellner introduced natural scientists to selected aspects of economic and econometric science, and employed Bayesian methods and entropy in economics and econometrics [22]. Hardin discussed living with in limits in ecology, economics, and population taboos [23]. Usually, entropy is applied mainly in ecological economics [24].

Chakrabarti, et al., researched econophysics and sociophysics [25]. Raine, et al., discussed new entropy law and the economic process [26]. Schwartzman studied the limits to entropy as the continuing misuse of thermodynamics in environmental and marxist theory [27]. Perrings, et al., searched irreversibility in economics [28]. De Pascale studied role of entropy in sustainable economic growth [29].

Econophysics and the law of entropy are foundations of economic phenomena. Rosser proposed ontological entropy as actual thermodynamic processes involved in the flow of energy from the Sun through the biosphere and economy, and metaphorical entropy as mathematics used for modeling entropy employed to model economic phenomena [30]. Areas considered include general equilibrium theory, growth theory, business cycles, ecological economics, urban-regional economics, income and wealth distribution, and financial market dynamics. The power-law distributions studied by econophysicists can reflect anti-entropic forces is emphasized to show entropic and anti-entropic forces can interact to drive economic dynamics, such as in the interaction between business cycles, financial markets, and income distributions.

In mainstream economics, markets are ideal if competition is perfect. When supply balances demand, economic maturity is orderly and disturbed only by shocks. James K. Galbraith and Jing Chen show that they contradict the foundations of scientific understanding of the physical and biological worlds. Entropy Economics discards the conventions of equilibrium and presents a new basis for thinking about economic issues. Their theory is based on scarcity, and accounts for the power of monopoly, whose production covers increasing and decreasing returns, uncertainty, fixed investments over time, and the impact of rising resource costs.

Together, their models illuminate key problems such as trade, finance, energy, climate, conflict, and demography [31].

The laws of energy and entropy are two basic laws, both are also very important quantities necessarily considered in the development of economy and society. But, entropy is often misunderstood in some cases. We proposed possible entropy decrease for isolated systems in natural and social sciences due to internal interactions [32-43] and transformation of internal energy. A universal formula for any isolated system is [33]:

$$dS = dS^a + dS^i, \quad (17)$$

where  $dS^a$  is an additive part of entropy and is always positive, and  $dS^i$  is an interacting part of entropy and can be positive or negative. Eq.(17) is similar to a well known formula:

$$dS = d_i S + d_e S, \quad (18)$$

in the theory of dissipative structure proposed by Prigogine.

New economics is connection to the information economy and network economy [44-47], and transforms commodity markets and society, and gives rise to a new economic order. In 1997, K. Kelly identified its three key characteristics: globality; prefer intangible ideas, information, and relationships; and high connectivity.

The premise of Post-Capitalism is sufficient: a vast array of goods become inexpensive or even free. The circular economy theory was first proposed by economist, philosopher, and poet Kenneth E. Boulding in 1966. In his book *The Economics of the Coming Spaceship Earth*, he likened Earth to a material-closed spaceship, advocating that resource recycling could alleviate environmental burdens and establish a closed-loop system of resources  $\rightarrow$  products  $\rightarrow$  recycled resources.

Based on the social structure we proposed the social individual-wave duality, which describes the social changes with a tendency after a tendency [48]. Economic development and prices in market economy have all wave property. The spectral density function can analyze and characterize economic fluctuations and economic cycles. In a word, in a circular economy and any cyclic process, entropy cannot increase always.

## VI. DEVELOPMENT CYCLE AND ECONOMIC CRISIS

N. Kondratieff proposed the long wave cycle of economy with 50 years [3], which must be the entropy decrease. Based on the nonlinear whole economics and the mathematical economics [49], we think that the nonlinear evolution is a universal rule for economic growth, and proposed the nonlinear theory of economic growth and its three laws [50]. It may combine the theorem of transformation from energy to quality on social development [51,52].

Further, the nonlinear theory of economic growth is applied to macroeconomics, if some unresolved issues exist, these will inevitably give rise to developmental cycles, such as the 50-year cycle [3]. Based on the nonlinear economics, my conclusion is that these cycles are not inherent and are variable: free competition leads to declining prices and profits, overproduction, and saturation in sectors like real estate, automobiles, and clothing, thereby ensuring the inevitability of developmental cycles and economic crises. Only through food, tourism, personal entertainment, and new directions we can continuously explore new possibilities.

If the political economy is an economy chaperoned polity, it will produce consequentially a binary economy. The political economy is usually imperfect economic question, even completely is not an economic question for some particular cases. It is not a strict economic rule, because in this case economy is only an appendage of polity. The economy will change along with polity.

The political economy as a multiply connected topological economics [53-56] can be described by the

complex function and the elliptic functions, which have two periods of economy and policy. The complex function corresponds to a complex system of economy and polity. In the multiply connected topological economy, economy corresponds to a real part, and policy and relation, etc., correspond to an imaginary part. A complex plane corresponds to the surface of the Riemann sphere, which is called a stereographic projection [57].

An elliptic function is a meromorphic function on the complex plane [57], and has the double periods:

$$f(z + m\omega + n\omega') = f(z). \quad (19)$$

Here  $\omega$  and  $\omega'$  are the two different basic periods. In this case, the economical development will possess two periods of economy and policy, respectively. It includes the Weierstrass elliptic function:

$$W(z) = \frac{1}{z^2} + \sum_{\omega \in L} \left[ \frac{1}{(z - \omega)^2} - \frac{1}{\omega^2} \right]. \quad (20)$$

Two systems and periods may be entangled.

In a multiply connected topological economy [53-56], political economics already exhibits a dual-cycle pattern. When particularly reckless or incompetent individuals seize power, politics severely disrupts the economy's inherent cycles. International cooperation faces challenges from rogue states.

## VII. APPLICATIONS AND SOME SUSTAINABLE DEVELOPMENT MODELS

We researched the nonlinear theory of economic growth [53,10], the multiply connected topological economics [53-56], the economic theory of knowledge economy [58,53,11], etc.

Some people argue that we are living in an era of utter decay, and a time when societal degeneration and the ugly aspects of human nature have expanded dramatically, and when various scammers and rogue elements run rampant. The era is entropy increase quick, and a time when an ever-increasing number of dictators, each one worse than the last, and a time when corrupt officials at all levels are becoming increasingly corrupt. Einstein said: "The search and striving for truth and knowledge is one of the highest of man's virtues." As the population grows ever larger, fewer people are pursuing truth, goodness, and beauty. There has been a trend around the world against equality, democracy and freedom, and try to build the realization of a quasi-authoritarian system. In the realms of science and education, prejudice is further from the truth than ignorance.

Mason proposed the Zero Project, whose framework is primarily composed of organizational units; these organizations are responsible for internal planning and allocation of goods, rather than being directly driven by market forces [3]. He proposed the five principles of transitions: 1. When confronting complex and fragile systems, it is essential to recognize the limitations of human willpower; networks outperform hierarchical structures in functionality. Network, complexity, fragility, and sensitivity are origin of nonlinear. 2. Ecological sustainability. 3. It is not merely a transition in economics; it must also be a transformation of humanity itself. 4. The ability to address issues comprehensively. 5. Maximizing informational power, which further involves the application of big data and AI to govern a digitally empowered democratic society. Principles 2-4 all reflect a whole approach. In summary, this is consistent with the concept of nonlinear whole economics.

Marc Edelman and Angelique Haugerud argued that, in the 21st century, there will be more than one type of globalization. They also highlighted the need for a new political economy [59]. Malcolm McIntosh questions the current model and mechanisms of capitalism and elaborates on five system changes that we need for a new political economy that is not based on neoliberalism and the commoditization and financialization of everything. He reprimands the blind pursuit of unsustainable growth and financial gains of neoliberal economics where

everything is financialized and says that this model “enslaves humanity” and “rapes the Earth” and stipulates that a new political economy is needed and must be based on systems thinking and incorporate the five system changes. He prognosticates a commercial world with the community as its central stakeholder featuring an interdisciplinary structure where science, philosophy, politics and economics coalesce while still considering community knowledge. He bases his ideas on five transitional, nascent, ineluctable, and necessary system changes [60].

We researched some types of economics from mathematics, nonlinearity, topology to synergetics and knowledge economics [61], and searched market economics guided by big data, and its basic characters are relations between big data and traditional economics and various economics [62].

We discussed some social sustainable developed patterns: 1) They may be the Lotka-Volterra model (Fig. 2), and correspond respectively to the three elements on Heaven-Man-Earth in Chinese traditional culture.

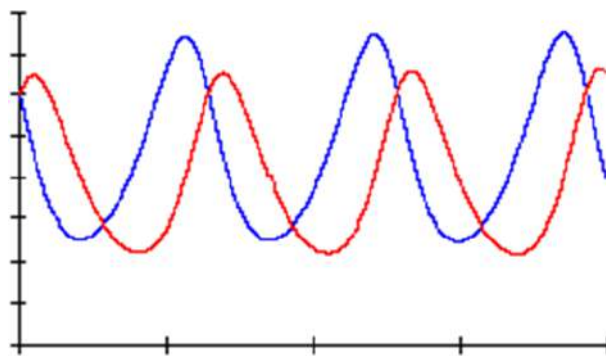


Fig. 2 Lotka-Volterra model

2) The synergetic pattern on society-economy-environment developed together.

3) The promotion-restraint pattern on Five-Elements. Combining Chinese traditional culture, we proposed the promotion-restraint sustainable developed pattern on the Five-Elements (Fig. 3), which include social (S) progress (democracy, justice, stabilization), economical development (F), science-technology (K), education-civilization (J) and environment-resource (H).

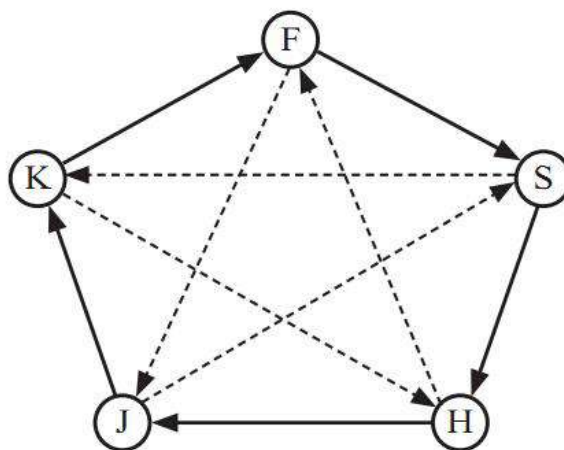


Fig.3 Promotion-Restraint Developed Pattern on the Five-Elements

Their basis is lower entropy, even entropy decrease. Natural laws are unfeeling, but world is not pessimistic always.

## VIII. SUMMARY

In certain respects, ecology is the foundation of economics; for instance, nonlinear whole ecology [5] corresponds to nonlinear whole economics, particularly in the context of various ecological crises, whether it concerns population pressure and living space, or environmental pollution.

In a word, economics is very complex, so we must study it by a variety of different methods.

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